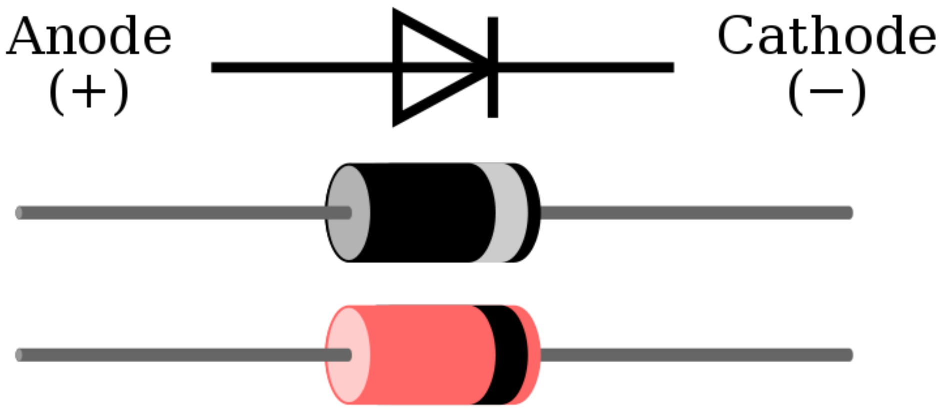
ELEC 2400 Electronic Circuits

Chapter 7: Diodes and Circuits



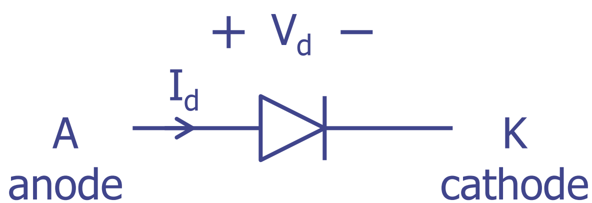
Course Website: https://canvas.ust.hk

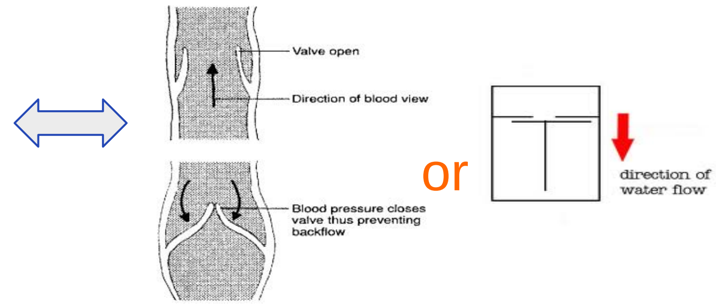
HKUST, 2020-21 Fall

Chapter 7: Diodes and Circuits

7.1 Ideal Diode

A diode conducts current in one direction, but blocks current in the reverse direction. The circuit symbol is



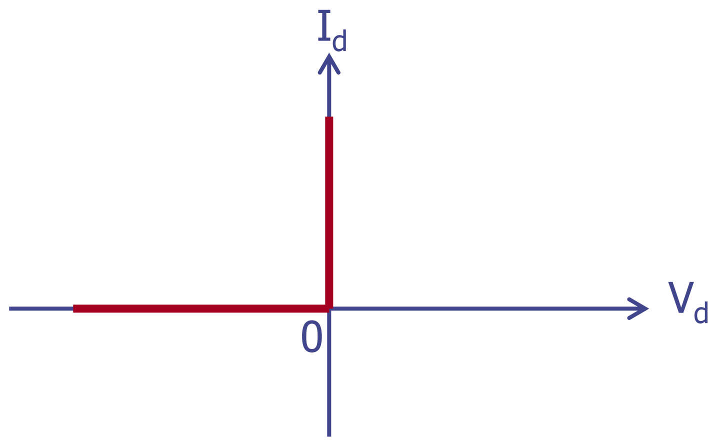


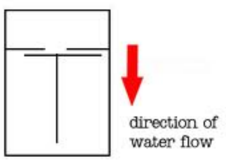
Current can only flow from the anode (+ve terminal) to the cathode (–ve terminal). The names were assigned for olden day vacuum tubes.

Examples of diodes:

7.1.1 I-V Characteristic of Ideal Diode

When a positive voltage is applied across an ideal diode, a current I_d flows from + to –. Note that the voltage across the diode V_d is 0. When a negative voltage is applied across it, $I_d = 0$. Hence, the I-V characteristic is





Clearly, the power dissipated by this device is always zero, and it is thus a passive component. Since it does not dissipate power, and it only conducts in one direction, it is a good choice to be a unidirectional switch.

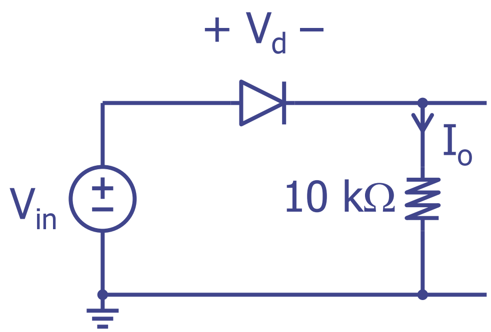
Ideal Diode Model

When the diode is forward biased ($V_d \geq 0$), the diode is ON and it behaves like a closed switch. Current flows through the diode.

When the diode is reverse biased ($V_d < 0$), the diode is OFF and it behaves like an open switch. No current flows.

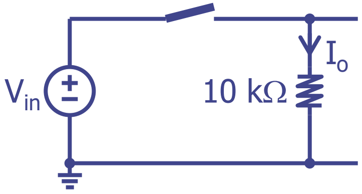
Example 7-1

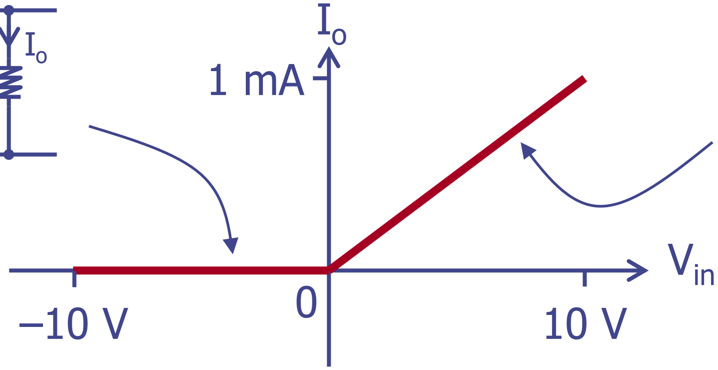
Example 7-1: Assume the diode is ideal, sketch I_o vs. V_{in} for V_{in} changes from –10 V to +10 V.

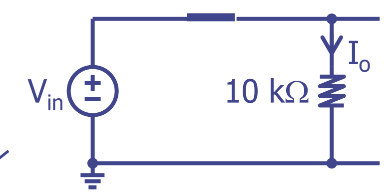


Soln.:

When V_{in}<0, I_o=0. When V_{in}>0, I_o=V_{in}/R (because V_d=0).





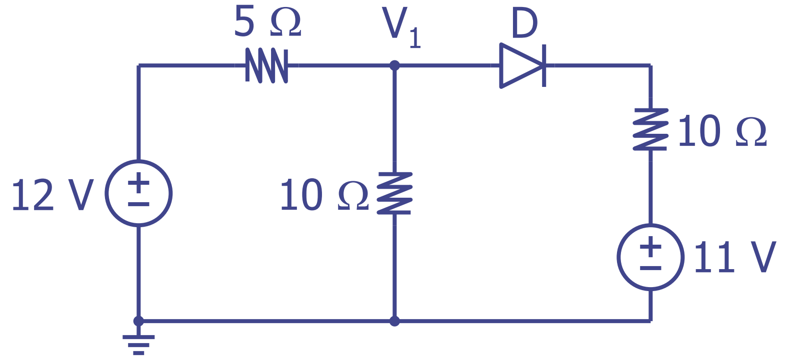


Analyzing Diode Circuits

To analyze a diode circuit:

(1) Assume the operating mode of the diode (either ON or OFF);
(2) Replace the diode with corresponding short or open circuit;
(3) Find relevant voltages and/or currents; and
(4) Check if the assumption in step (1) is correct or not. If it is correct, the analysis is complete. If it is not correct, make another assumption in step (1) and repeat steps (1) to (4)

Example 7-2: Assume the diode is ideal, find V_1.



Example 7-2 (cont.)

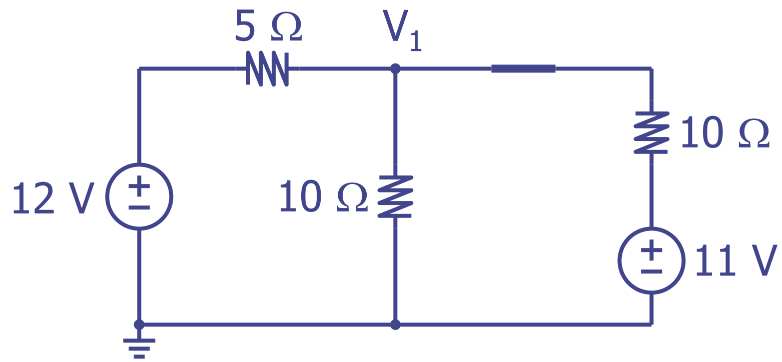
Soln.:

Case 1: Assume the diode is ON, and replace D with a short:

$$\frac{12-V_1}{5} = \frac{V_1}{10} + \frac{V_1 - 11}{10}$$

$$\Rightarrow \quad 2.4 + 1.1 = \frac{V_1}{10} + \frac{V_1}{10} + \frac{V_1}{5}$$

$$\Rightarrow \quad V_1 = 8.75V$$

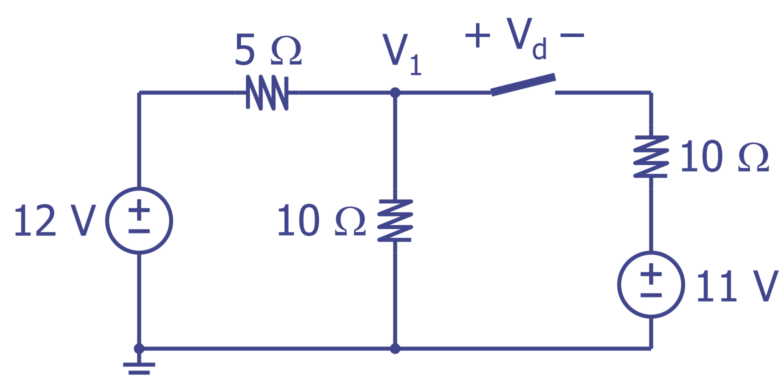


However, if V_1=8.75 V, diode current would flow backward. So D should be OFF instead.

Case 2: Assume the diode is OFF, and replace D with an open:

$$V_1 = \frac{10}{5+10} \times 12$$

$$= 8V$$



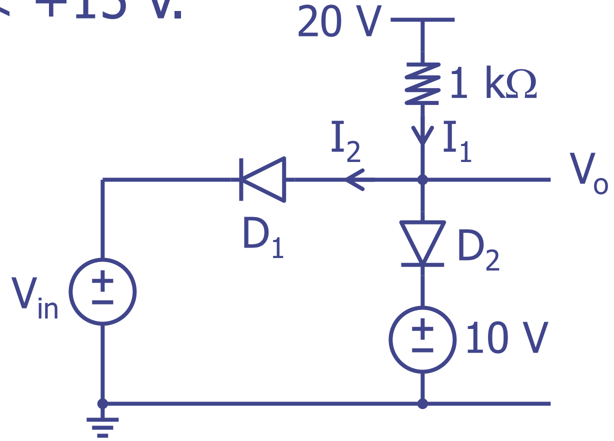
Moreover,

$$V_d = 8 - 11$$

$$= -3V$$

Example 7-3

Example 7-3: Assume the diode is ideal, plot I_1 vs. V_{in} and I_2 vs V_{in} for $0 < V_{in} < +15$ V.



Soln.:

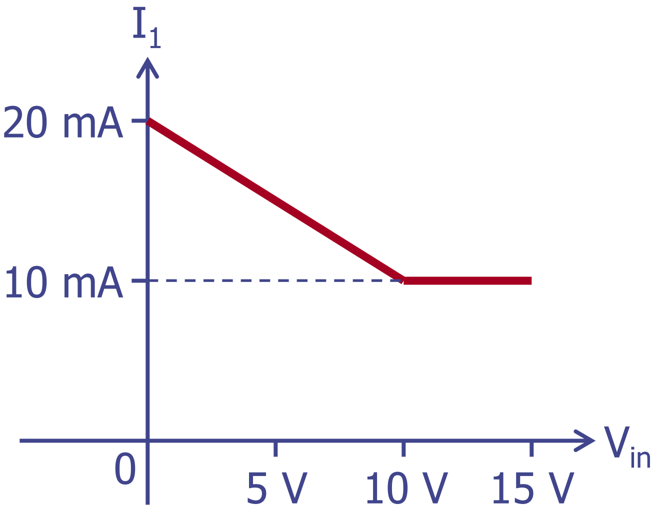
(1) I_1 vs. V_{in}:

For $0<V_{in}<10$ V, D_1 is ON, D_2 is OFF:

$$I_1 = \frac{20 - V_{in}}{1k}$$

For $V_{in}>10$ V, D_1 is OFF, D_2 is ON:

$$I_1 = \frac{20-10}{1k} = 10mA$$



Example 7-3 (cont.)

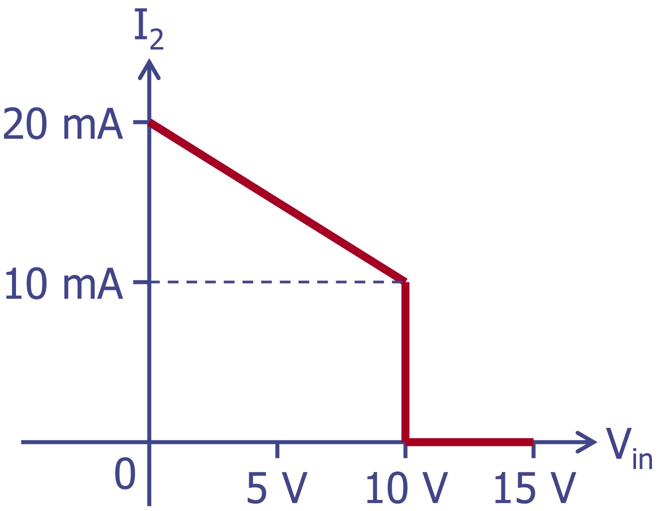
(2) I_2 vs. V_{in}:

For $0<V_{in}<10$ V, D_1 is ON, D_2 is OFF:

$$I_2 = I_1 = \frac{20 - V_{in}}{1k}$$

For $V_{in}>10$ V, D_1 is OFF, D_2 is ON:

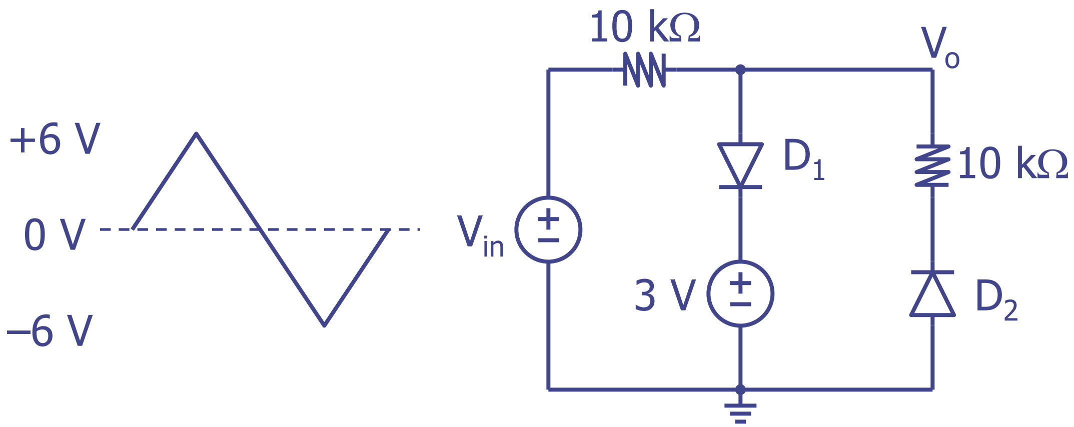
$$I_2 = 0mA$$



7.1.2 Diode Clipping Circuit

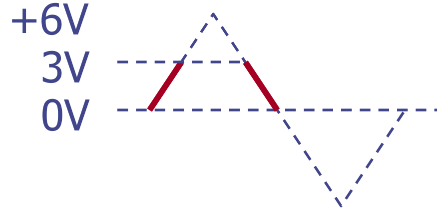
A diode circuit can be used to reshape or clip the input waveform.

Example 7-4: Assume the diode is ideal, plot V_o w.r.t. V_{in}.



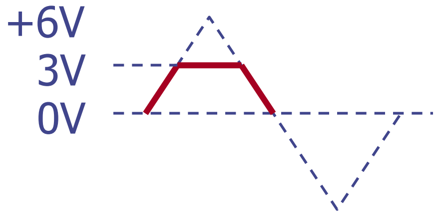
Soln.:

(1) For $0<V_{in}<3$ V, both D_1 and D_2 are OFF, hence, $V_o = V_{in}$.



Example 7-4 (cont.)

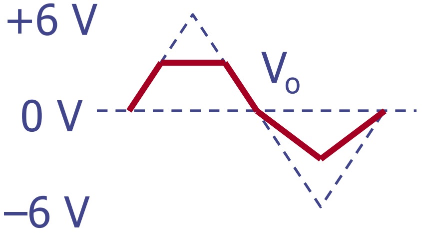
(2) For $3<V_{in}<6$ V, D_1 is ON and D_2 is OFF, hence, $V_o = 3$ V.



(3) For -6 V$<V_{in}<0$ V, D_1 is OFF and D_2 is ON, hence,

$$V_o = \frac{10k}{10k+10k} V_{in} = \frac{V_{in}}{2}$$

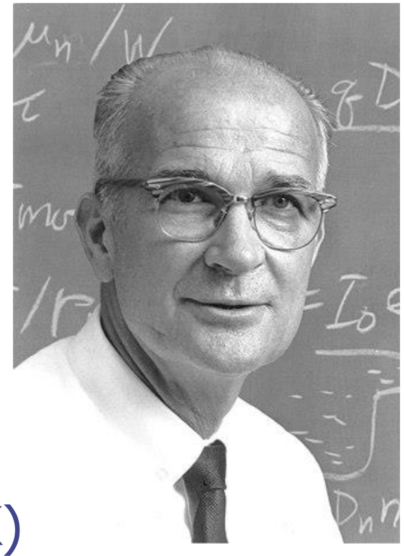
The final plot is



7.2 Physical Diodes

A physical diode, of course, behaves differently. From semiconductor physics, the terminal characteristic of the diode is described by William Shockley's diode equation (I-V characteristic):

$$I_d = I_s\left(e^{V_d/V_T} - 1\right)$$



William Shockley Jr.
1910 - 1989

where

I_s = reverse saturation current
V_T = kT/q = thermal voltage
k = Boltzmann's constant = 1.38×10^{-23} J/K
T = absolute temperature in Kelvin (27 ^oC $\approx$ 300 K)
q = electronic charge = 1.6×10^{-19} C

For manual computation, we usually take $V_T(27\ ^oC) = V_T(300\ K)$ = 26 mV or 25 mV.

https://en.wikipedia.org/wiki/William_Shockley

Co-recipient of the Nobel Prize in Physics for the invention of the transistor

7.2.1 Forward and Reverse Bias

When $V_d>0$, the diode is said to be forward biased. Especially when $V_d>3V_T$, the -1 term in the diode equation can be neglected, and I_d increases exponentially w.r.t. V_d:

$$I_d \approx I_s e^{V_d/V_T}$$

When $V_d=0$, the diode is zero biased. With $e^0 = 1$, we have

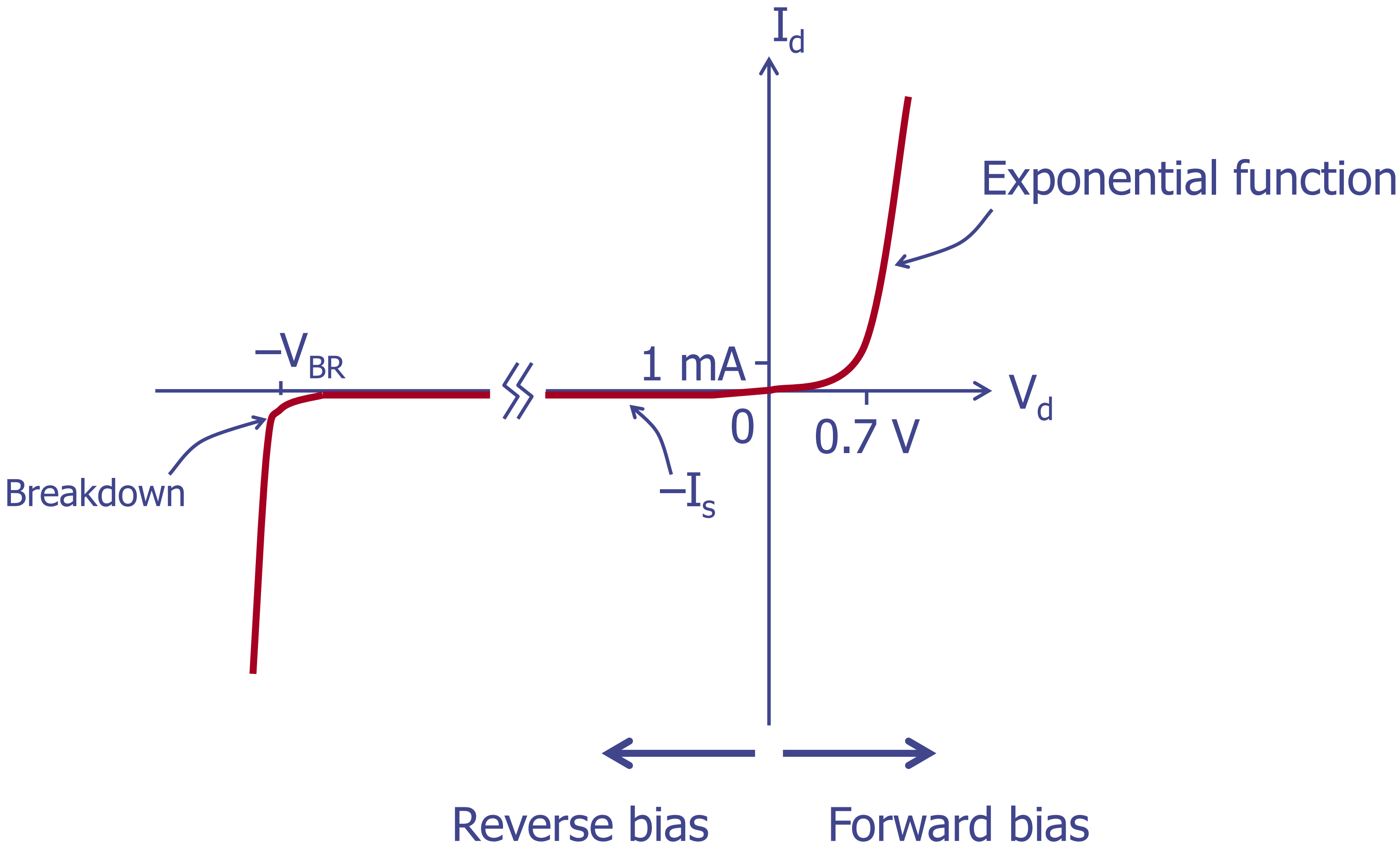
$$I_d = 0$$

When $V_d<0$ the diode is reverse biased. When $V_d<-3V_T$, the exponential term becomes too small when compared to 1 and can be neglected. The diode current "saturates" at the small value of $-I_s$:

$$I_d = -I_s$$

However, when V_d is large negative ($V_d = -V_{BR}$), the diode will undergo reverse breakdown, and large (negative) current can flow through the diode.

7.2.2 I-V Characteristic of Physical Diode



1-mA Diode

For a tiny diode on an integrated circuit (IC), the reverse saturation current I_s is very small, e.g., $I_s = 2\times10^{-15}$ A = 2 fA. To pass 1 mA through the diode, the voltage drop is:

$$
\begin{aligned}
& 1\text{mA} && = 2\times10^{-15} e^{V_d/V_T} \\
\Rightarrow \quad & V_d && = 25.88\text{m}\times\ln\frac{1\text{m}}{2\times10^{-15}} \\
& && = 0.697 \approx 0.7\text{V}
\end{aligned}
$$

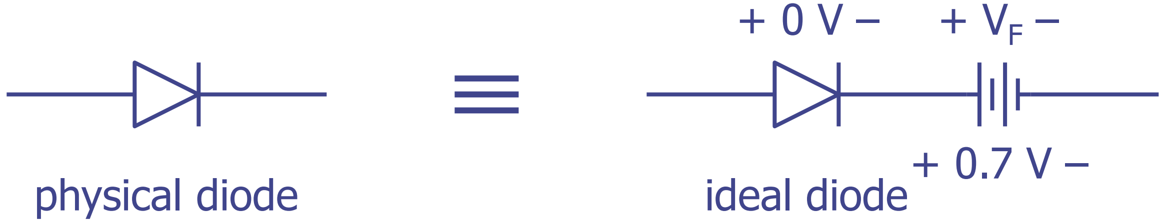
This is called a 1 mA diode, in the sense that it conducts a current of 1 mA when the diode voltage drop V_d = 0.7 V.

For a 10 mA diode, I_d = 10 mA at V_d = 0.7 V, and

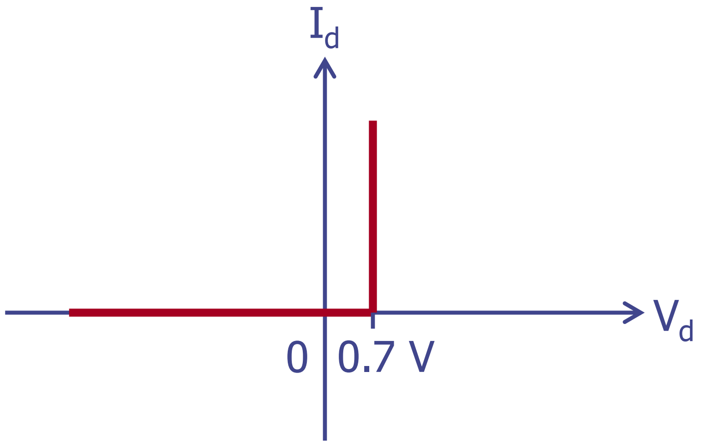
$$I_s = 2\times10^{-14}\text{A} = 20\text{fA}$$

7.2.3 Offset Diode Model and Piecewise Linear Diode Model

For a 1 mA diode, the diode drop is V_d=0.7 V at I_d=1 mA. Different diodes will have a different I_s, and thus, a different V_d. However, as a rule of thumb, we use V_F=0.7 V in modeling diodes. That is, we model the physical diode as an ideal diode in series with a battery of 0.7 V when forward biased. The model is called the Offset Diode Model.

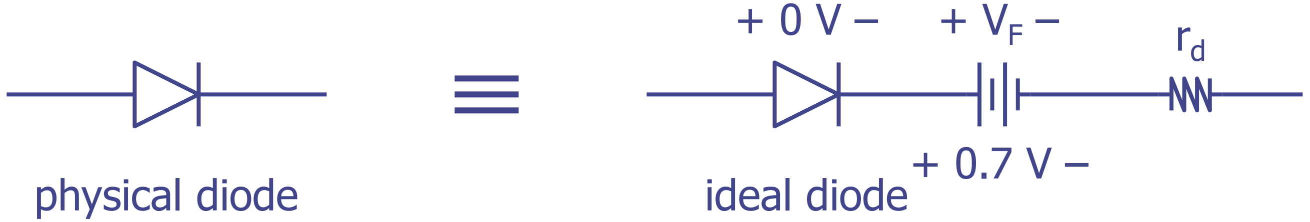


I-V characteristic:

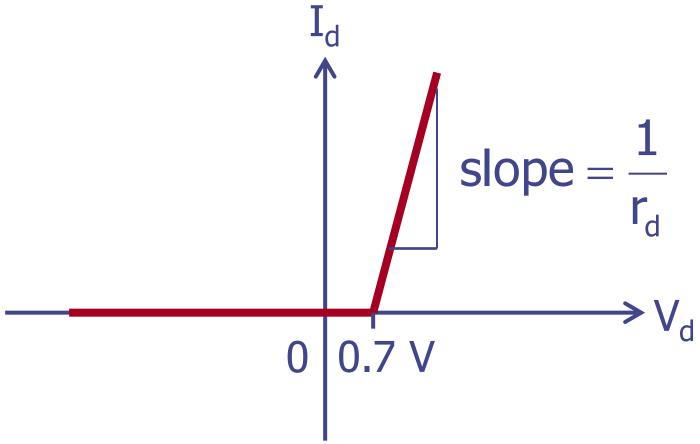


Piecewise Linear Diode Model

In reality, a diode will have a non-zero diode resistance when it is forward biased. The diode characteristic can be approximated by a two-segment piecewise linear model, named as Piecewise Linear Diode Model (PWL model). Typically, the sloped line segment would be chosen tangent to the diode curve at a particular quiescent point, e.g. 1 mA.

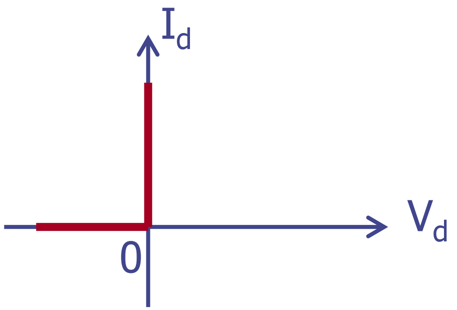


I-V characteristic:

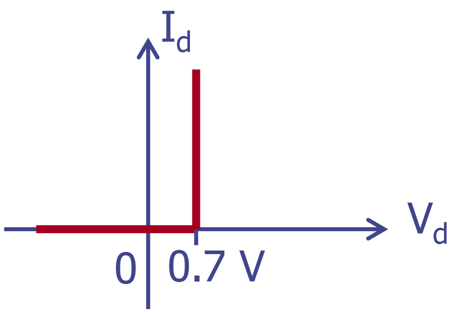


Diode Models in Forward Biased

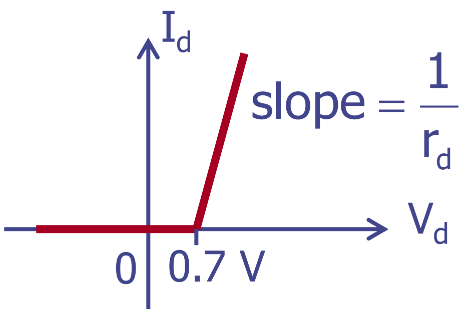
Ideal Diode Model



Offset Diode Model

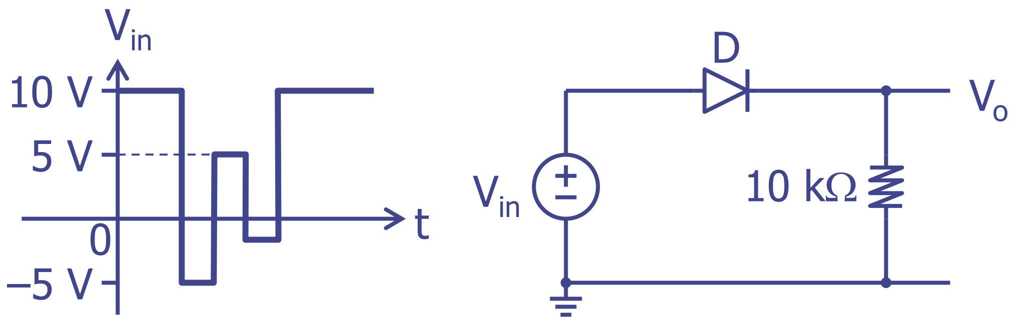


Piecewise Linear (PWL) Diode Model



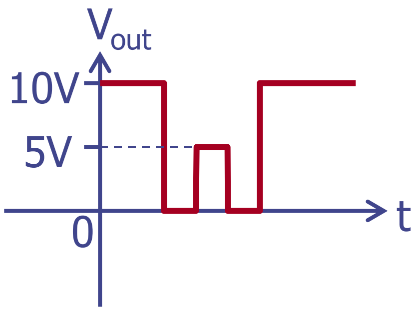
Example 7-5

Example 7-5: Sketch V_o assuming that the diode (1) is ideal; and (2) is modeled with a forward diode drop of 0.7 V; (3) is modeled with a PWL model with V_F=0.7 V and r_d=100 Ω.

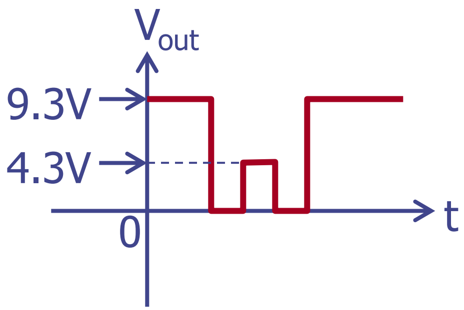


Soln.:

(1)



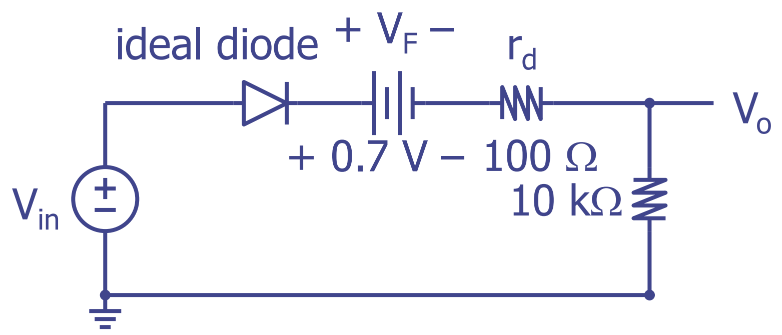
(2)



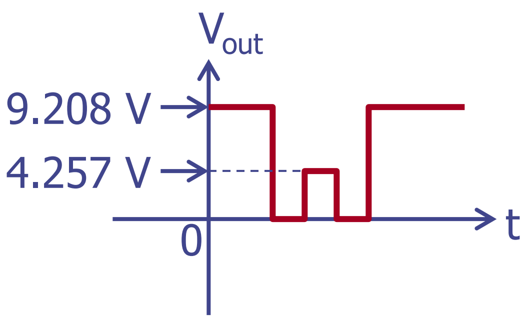
Example 7-5 (cont.)

Soln.:

(3) When V_{in}>0.7 V, the diode turns on and the diode can be replaced by its PWL model.



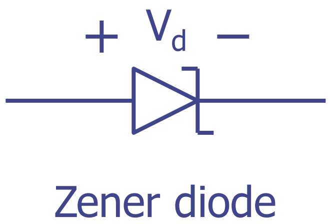
$$V_o = \frac{10k}{10k + 100}(V_{in} - 0.7)$$

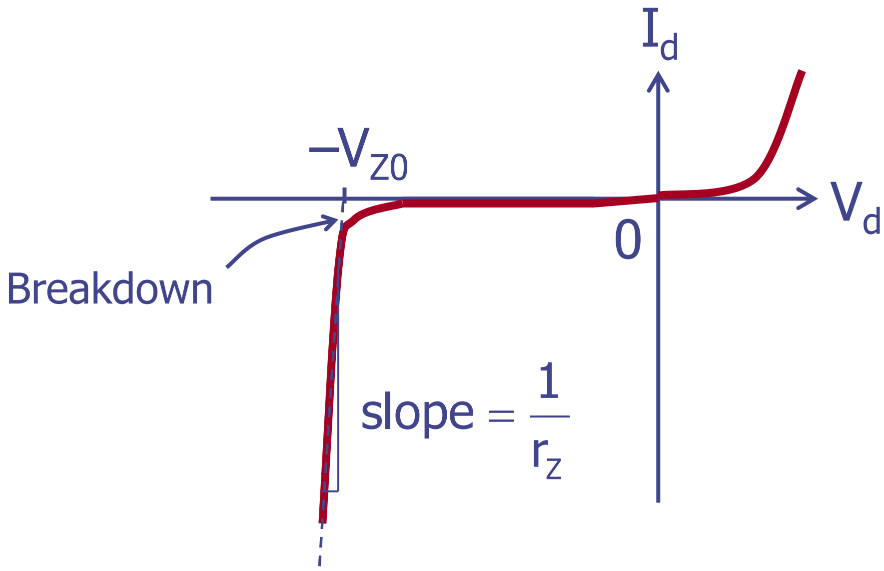


When V_{in}=10 V, V_o=9.208 V

When V_{in}=5 V, V_o=4.257 V

7.3 Zener Diode



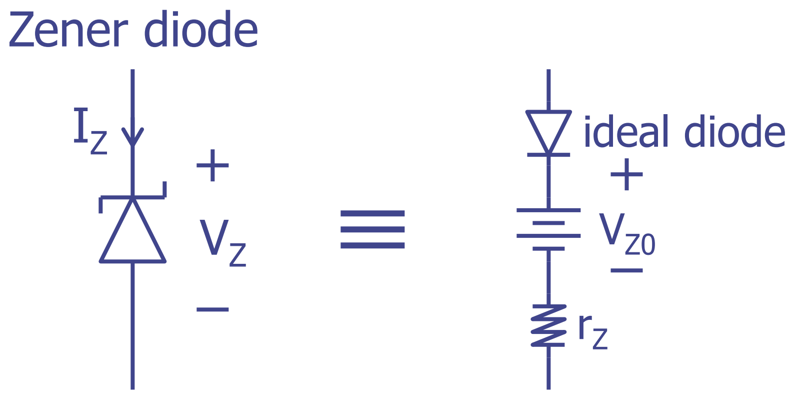


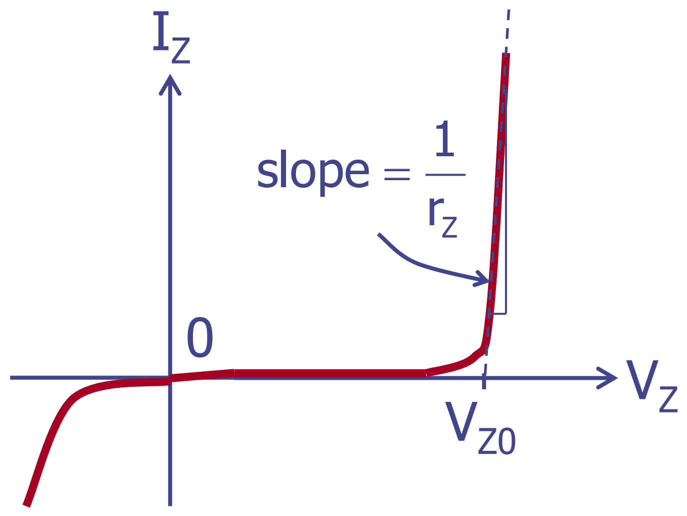
In the breakdown region, the reverse biased I-V characteristic has a very steep slope.

In this region, the voltage across the diode remains nearly constant while the current varies (i.e. a very small internal resistance).

A Zener diode is designed to operate in the breakdown region.

Zener Diode Circuit Model



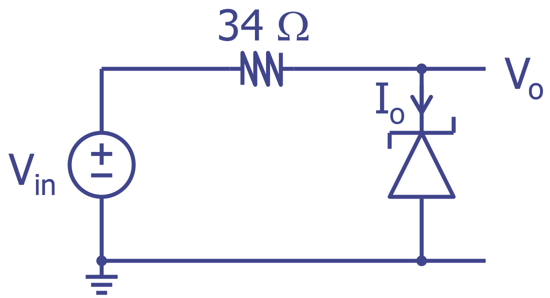


The Zener diodes are usually operated in the reverse bias region, and a PWL model can be used to model the diode characteristic such that $V_Z = V_{Z0} + I_Z r_Z$.

They are usually used to produce constant output voltages.

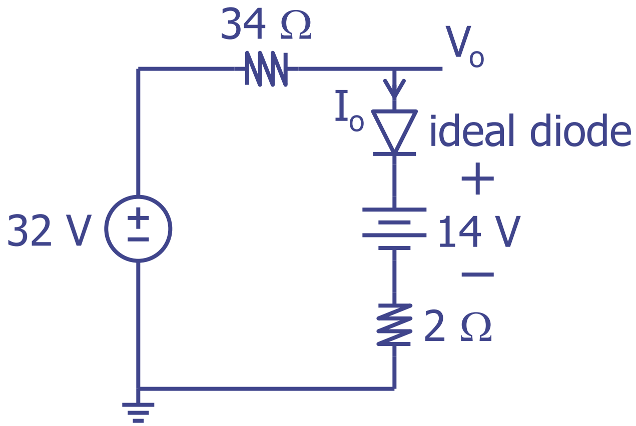
7.3.1 Zener Diode as Voltage Regulator

Example 7-6: Given that V_{Z0}=14 V and r_Z=2 Ω, find I_o and V_o when (1) V_{in}=32 V and (2) V_{in}=35.2 V.



Soln.:

(1)



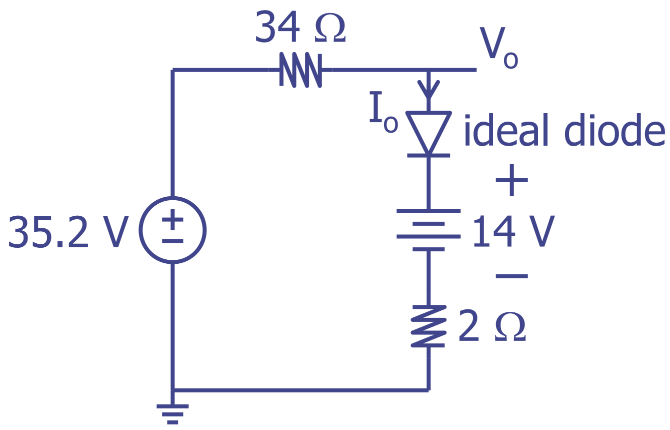
$V_{in} > V_{Z0}$, Zener diode is in breakdown region.

$$I_o = \frac{32V - 14V}{34\Omega + 2\Omega} = 0.5A$$

$$V_o = 14V + I_o \times 2\Omega$$
$$= 14V + 0.5A \times 2\Omega = 15V$$

Zener Diode as Voltage Regulator (cont.)

(2)



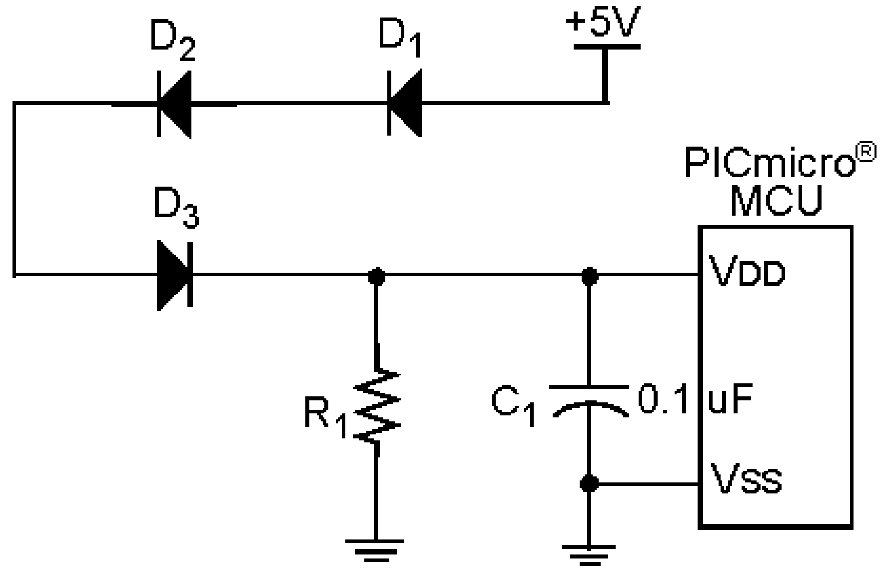
$$I_o = \frac{35.2V - 14V}{34\Omega + 2\Omega} = 0.589A$$

$$V_o = 14V + I_o \times 2\Omega$$
$$= 14V + 0.589A \times 2\Omega = 15.18V$$

When the input voltage changes by 10% (32 V $\rightarrow$ 35.2 V), the output voltage only changes by 1.19% (15 V $\rightarrow$ 15.18 V).

V_o is roughly a constant voltage. This circuit can regulate the output voltage to almost a constant value despite the change of the input voltage.

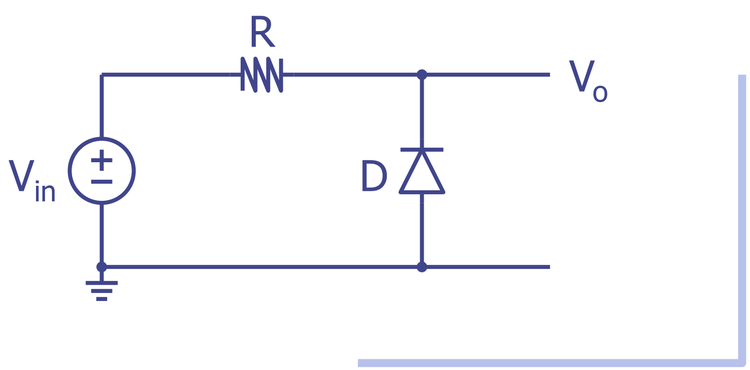
7.4 Diode Circuit Applications

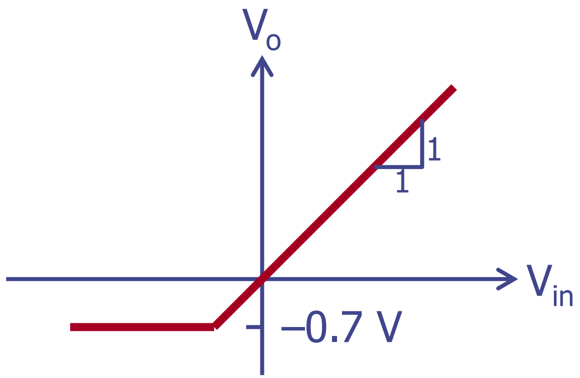


7.4.1 Limiter and Clipper

A limiter (clipper, clamping circuit) is used to confine the output voltage within some boundaries.

Example 7-7: Sketch V_o assuming that the diode has a forward diode drop of 0.7 V.



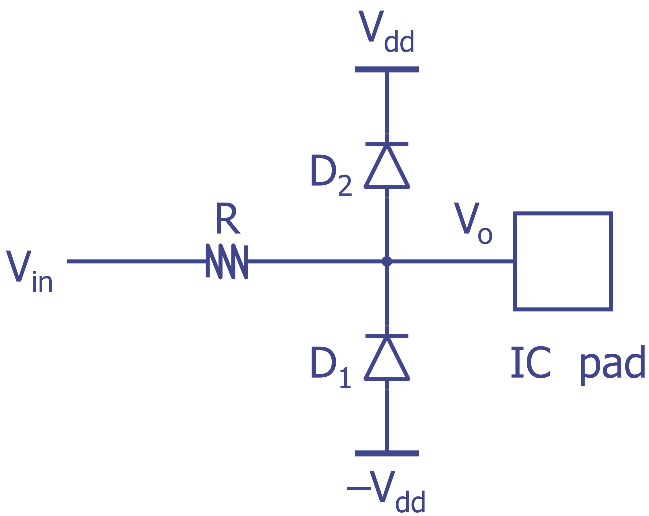


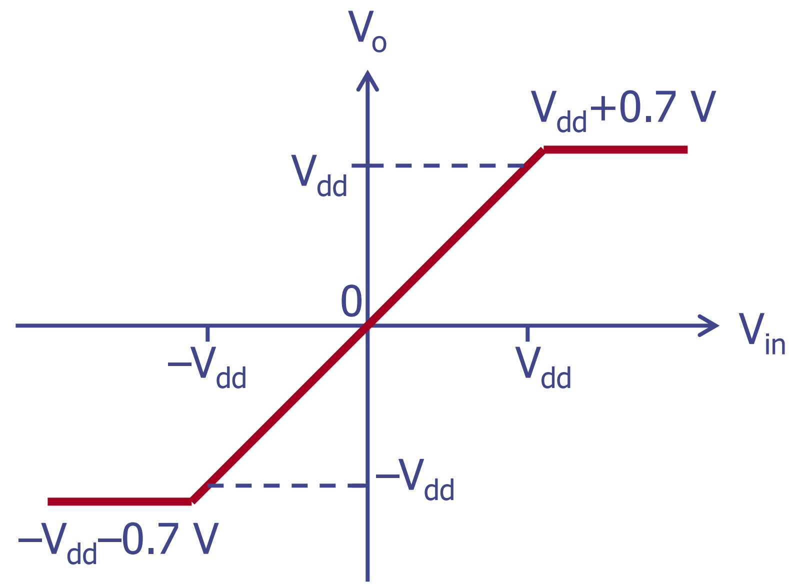
Soln.:

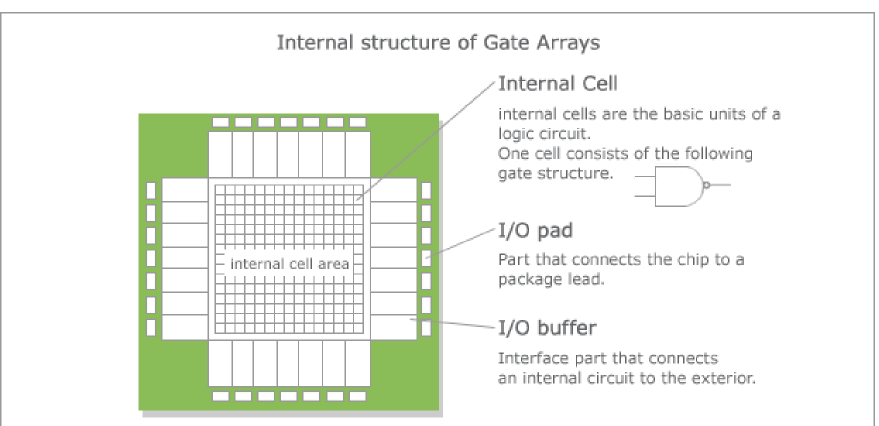
When $V_{in}>0$, the diode is reverse biased, and $V_o = V_{in}$. But when $V_{in}<0$, especially when $V_{in}<-0.7$ V, the diode is forward biased, and current flows through the diode D and the resistor R, and into V_{in}. The output is thus limited at –0.7 V.

Limiter for I/O Pad

A more useful limiter is used to prevent a node from being damaged by swinging too high or too low, e.g., the I/O pad of an integrated circuit.

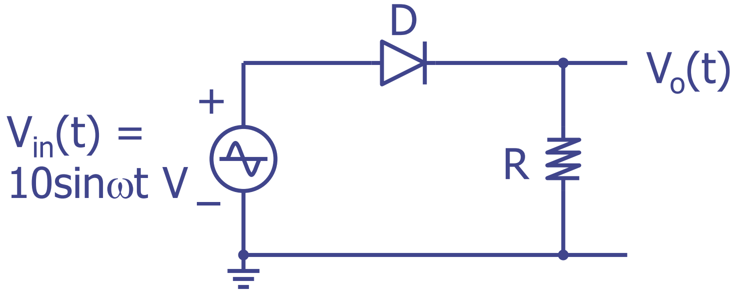




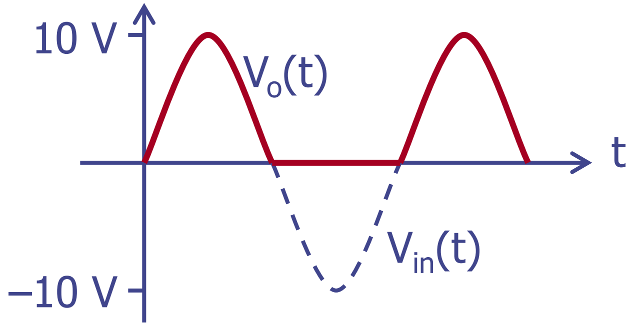


7.4.2 Half-Wave Rectifier

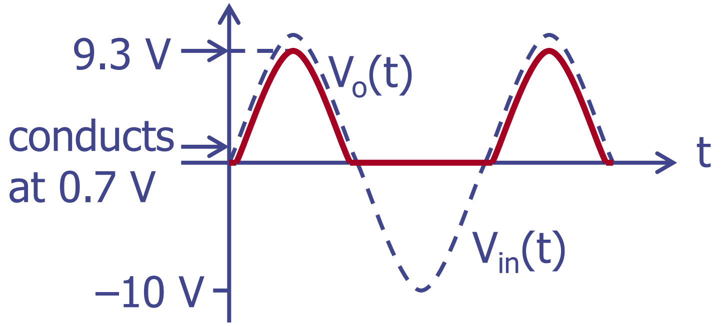
A diode only conducts in one direction, and it can be used to rectify an AC signal into a signal with only one polarity. The simplest rectifying circuit is the half-wave rectifier, with only half of the cycle (the positive cycle) appears at the output.



Rectifier with ideal diode:

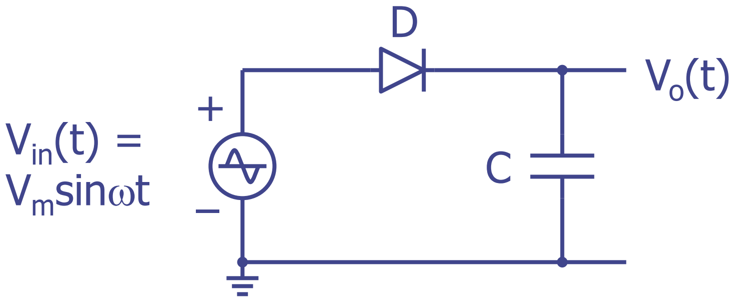


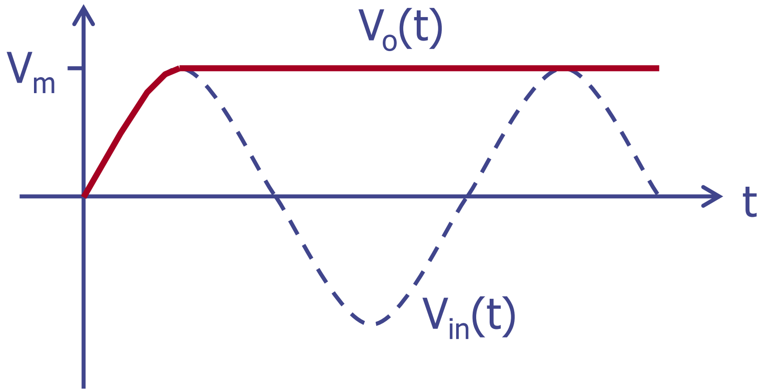
Rectifier with physical diode:



AC to DC Rectifier

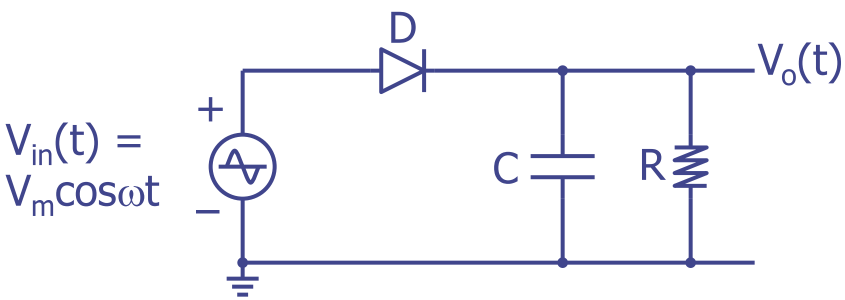
Although the half-wave rectifier converts a bipolar voltage (AC) into a uni-polar voltage, most electronic circuits need a constant DC voltage as the power supply. To turn the rectified sine wave into a DC voltage, we may replace the resistor R with a capacitor C. Assume the diode is ideal and the capacitor is initially relaxed, let us sketch the output voltage $V_o(t)$.

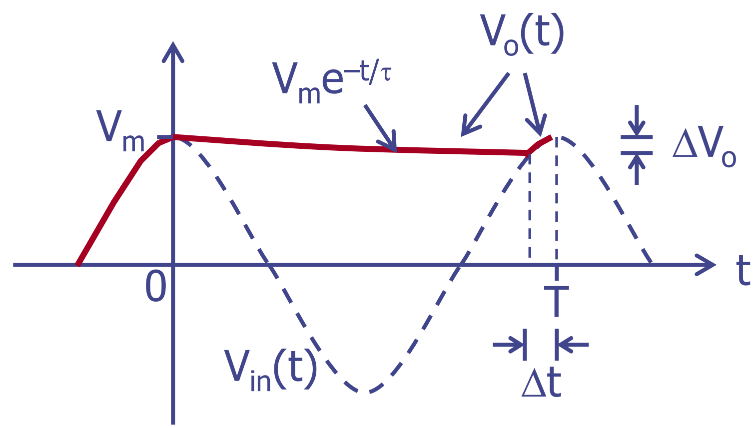




Rectifier with Load

The DC output voltage V_o can now be used as the power supply of electronic circuits that needs a DC power supply. The load can be modeled as a resistor R or a current source I_o (= V_o/R). However, the load current discharges the capacitor C and the output voltage drops when C is not charging up by $V_{in}(t)$.



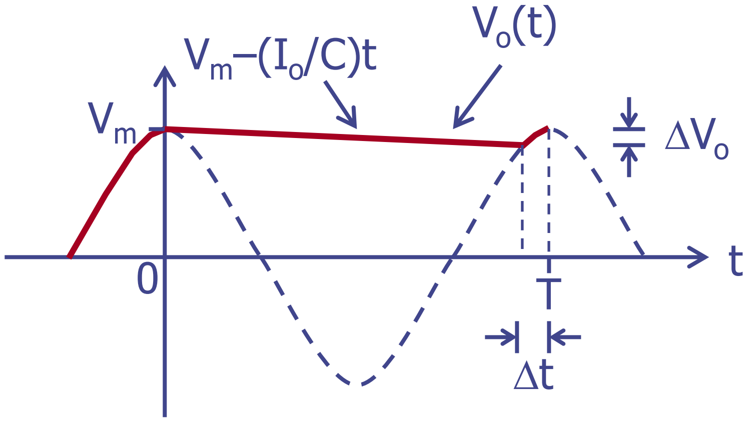


The input voltage $V_{in}(t)$ charges up C again when $V_o(t)$ tends to drop below $V_{in}(t)$ at t = T–Δt.

Voltage Ripple of Rectifier

To avoid dealing with an exponential function, let the resistive load R be replaced by the load current I_o (=V_o/R). When $V_{in}(t)$ falls below $V_o(t)$, the diode is reverse-biased, and $V_o(t)$ drops because C is discharged by I_o:

$$V_o(t) = V_m - \frac{I_o}{C} t$$



The diode will be forward-biased again when

$$V_o(T - \Delta t) = V_m \cos(\omega(T - \Delta t))$$

That is, we need to solve

$$V_m - \frac{I_o}{C}(T - \Delta t) = V_m \cos(\omega(T - \Delta t))$$

The output voltage ripple ΔV_o is given by

$$\Delta V_o = \frac{I_o}{C}(T - \Delta t)$$

Output Voltage of Rectifier

Solving for $t=T-\Delta t$ to obtain ΔV_o can be done numerically, but observe that the output voltage ripple ΔV_o should be very small, say, 1% of the average output voltage V_o, such that $\Delta t << T$. Therefore,

$$\Delta V_o \approx \frac{I_o T}{C} \qquad \left(= \frac{V_o T}{CR} \right)$$

The average output voltage V_o is given by

$$V_o = \frac{V_o(0) + V_o(T - \Delta t)}{2}$$

$$\approx V_m - \frac{I_o T}{2C} \qquad \left(\approx V_m - \frac{V_m T}{2CR} \right)$$

In fact, it is sometimes acceptable to just take $V_o = V_m$.

Example 7-8

Example 7-8: The AC voltage of the wall socket is $V_{in}(t) = 311\cos(2\pi 50t)$ V. If ΔV_o is 1% of V_o, compute the value of the capacitor needed if the load current is 1 A.

Soln.:

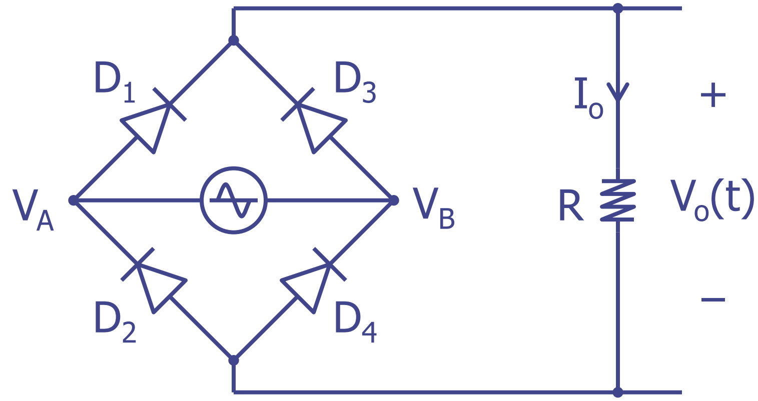
$$\Delta V_o \approx \frac{I_o T}{C}$$

$$\Rightarrow \quad C = \frac{I_o T}{\Delta V_o} = \frac{1 \times 20\text{m}}{0.01 \times 311}$$

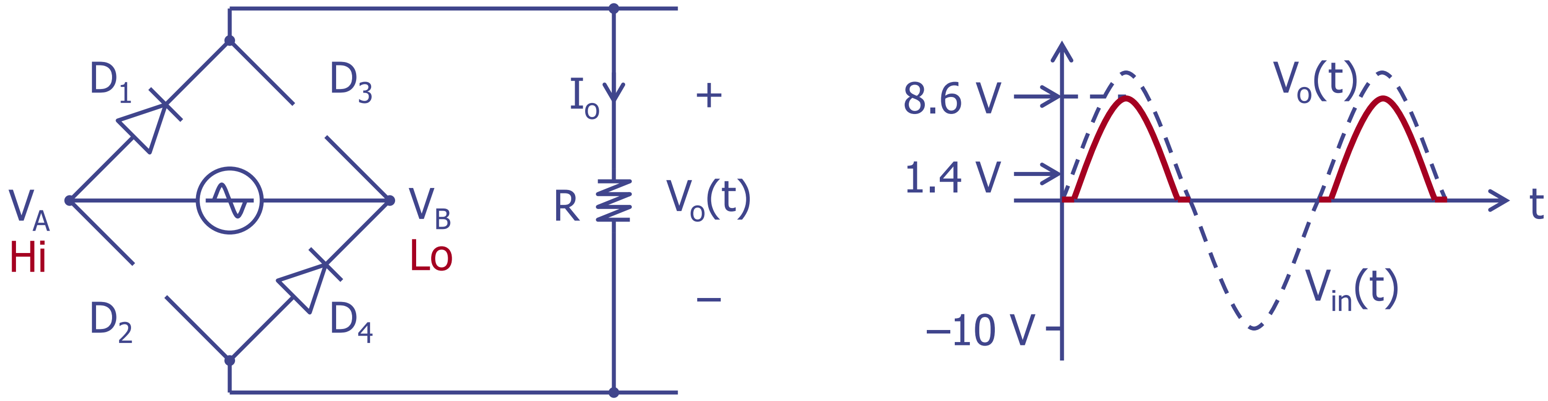
$$= 6430\mu\text{F}$$

7.4.3 Full-Wave Rectifier

A full-wave rectifier using four diodes in a bridge connection can convert even the negative cycle to be positive. This full-wave rectifier is also known as a diode bridge.

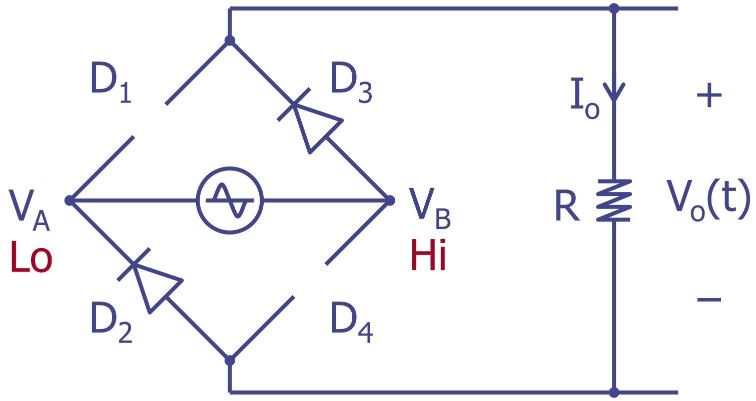


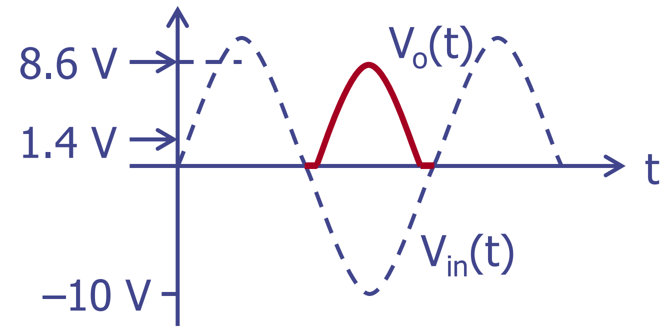
When $V_A > V_B$, the current flows from V_A, through D_1, then R, then D_4 and V_B to complete the cycle.



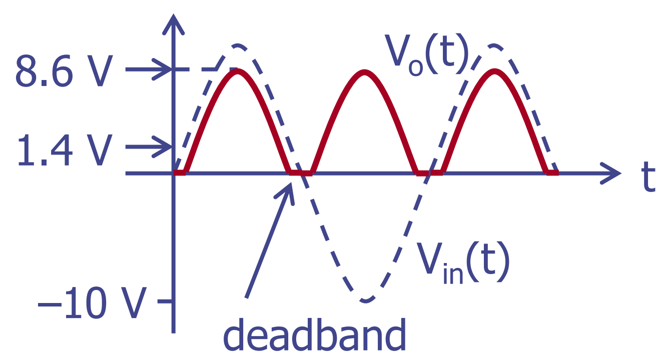
7.4.3 Full-Wave Rectifier (cont.)

When $V_B > V_A$, the current flows from V_B, through D_3, then R, then D_2 and V_A to complete the cycle.





In both cases, the current passes through R in the same direction (through two diodes).

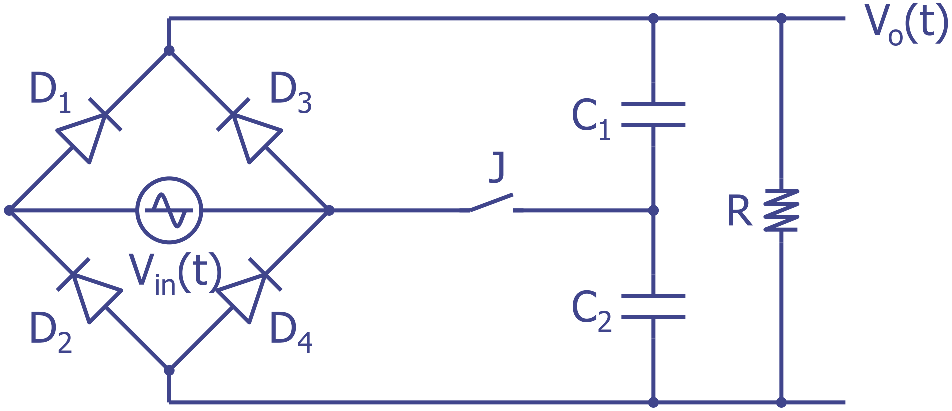


7.4.4 Universal Rectifier

For household electrical appliances, Hong Kong and the United States have different standards:

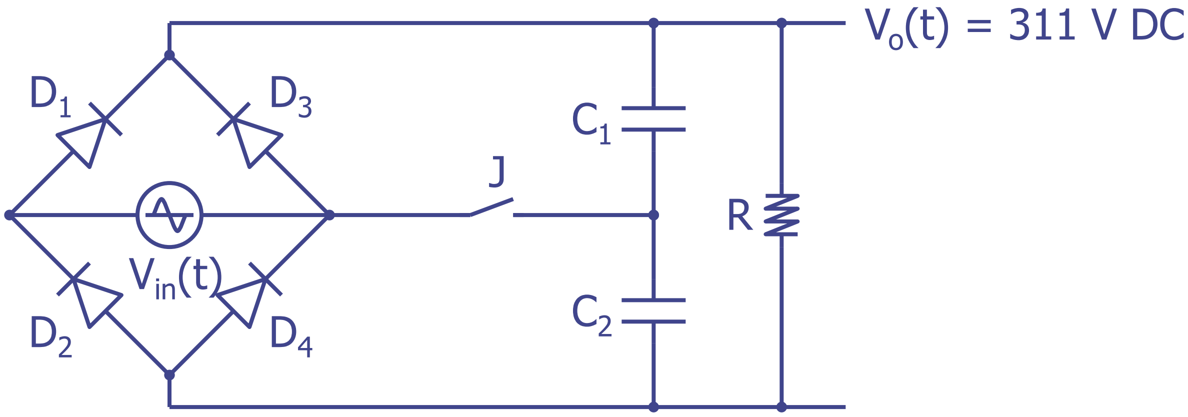
Hong Kong:	V_{rms}	$= 220$ V, $f = 50$ Hz
US:	V_{rms}	$= 110$ V, $f = 60$ Hz

In order for the power supply (usually a DC-DC switching converter) of the appliance to use both V_{rms}'s, a universal rectifier is used.



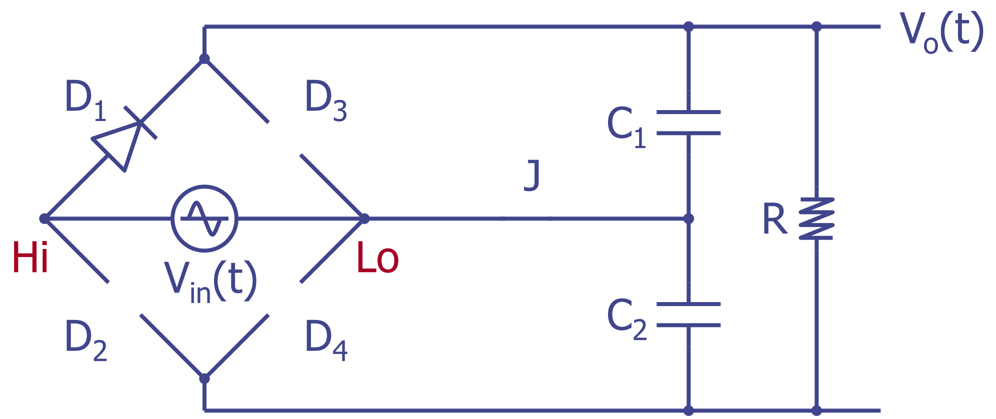
Universal Rectifier: Principle of Operation

When V_{rms} = 220 V, the jumper J is opened, and a regular full-wave rectifier is obtained, with $V_o(t) \approx V_p$ ($= 220\sqrt{2} = 311$ V DC).

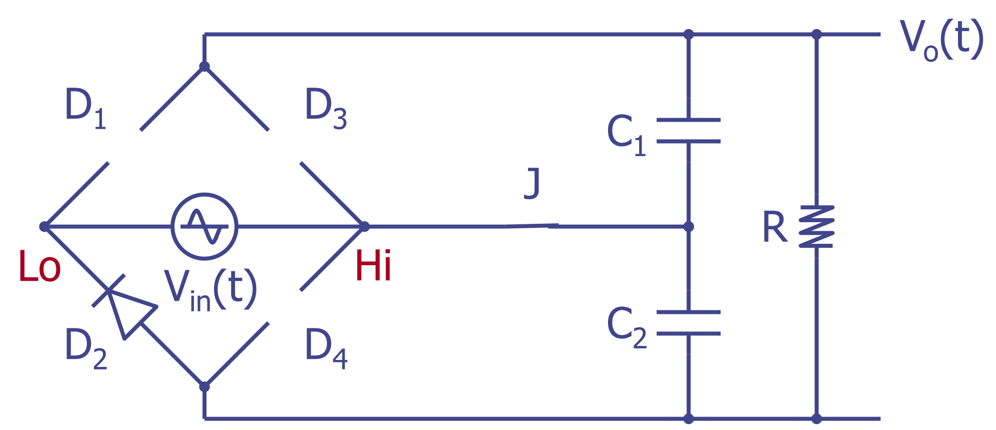


Universal Rectifier: Principle of Operation (cont.)

When V_{rms} = 110 V, the jumper J is closed. Then $V_{in}(t)$, D_1 and C_1 together form a half-wave rectifier that conducts for $V_{in}(t) > 0$, and $V_{c1}(t) \approx V_p$ ($= 110\sqrt{2}$ = 155.5 V DC).

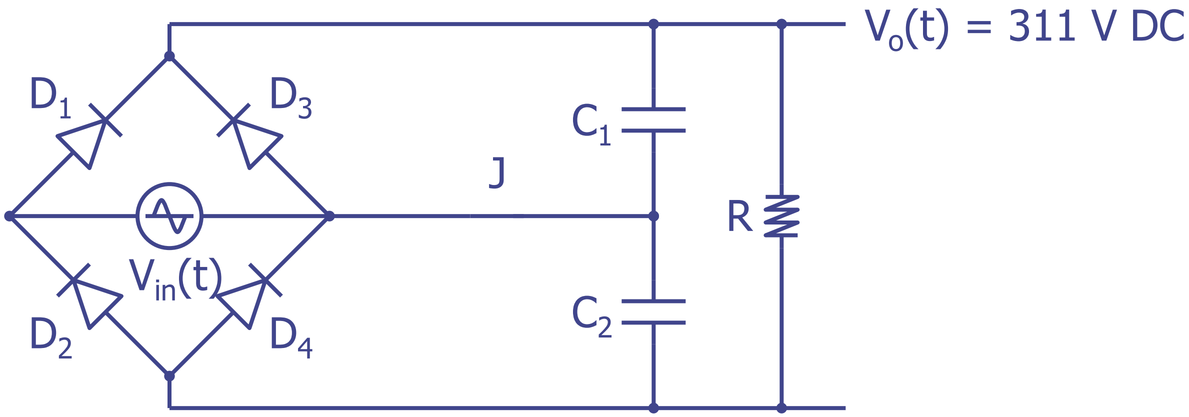


Also, $V_{in}(t)$, D_2 and C_2 together form another half-wave rectifier that conducts for $V_{in}(t) < 0$, and $V_{c2}(t)$ = 155.5 V DC.



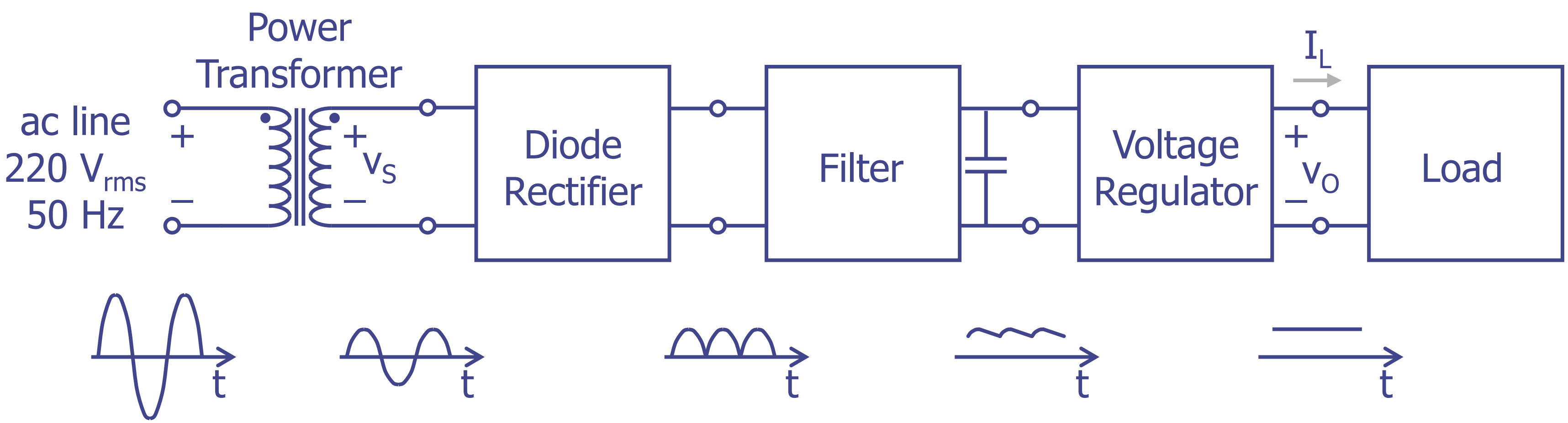
Universal Rectifier: Principle of Operation (cont.)

Now, $V_o(t) = V_{c1}(t) + V_{c2}(t) = 311$ V DC, so the output voltage is the same as the case when $V_{rms} = 220$ V.



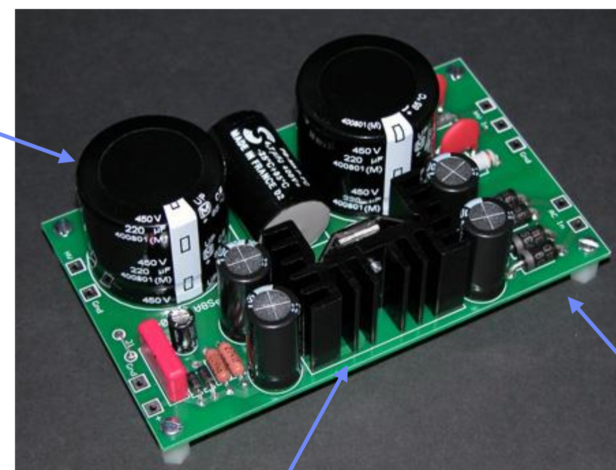
For older appliances, the jumper is a mechanical component, and the manufacturer will make appropriate connection before shipping to different countries. For computers and mobile phones, all adaptors + chargers can detect the input voltage automatically and determine if the relay (electrical jumper) should be open or close.

AC to DC Power Supplies



Power
Transformer

Filter

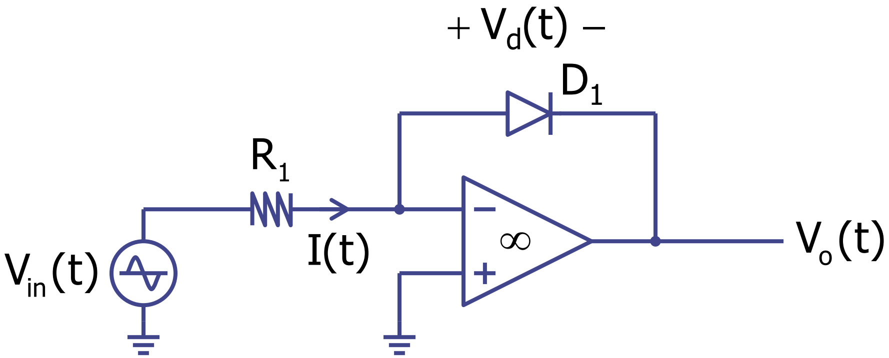


Voltage
Regulator

Diode
Rectifier

7.5.1 Logarithmic Amplifier

By connecting a diode in the feedback path of an op amp, a logarithmic amplifier can be realized.

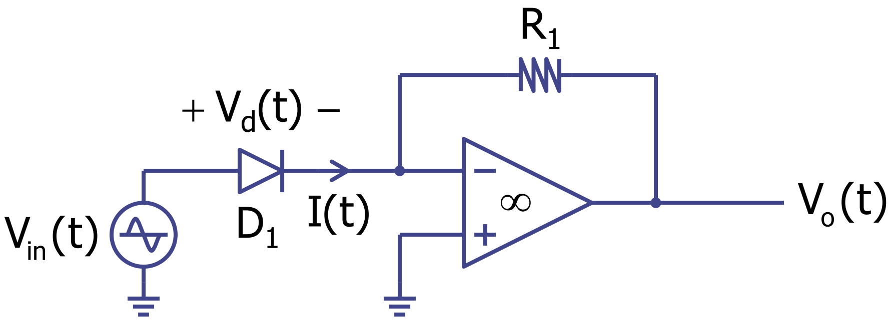


$$\frac{V_{in}(t)}{R_1} = I(t) = I_S\left(e^{V_d(t)/V_T} - 1\right) \approx I_S e^{V_d(t)/V_T}$$

$$\Rightarrow \quad V_o(t) = -V_d(t) = -V_T \ln \frac{V_{in}(t)}{R_1 I_S}$$

7.5.2 Exponential Amplifier

Similarly, an exponential (anti-logarithmic) amplifier can be realized.

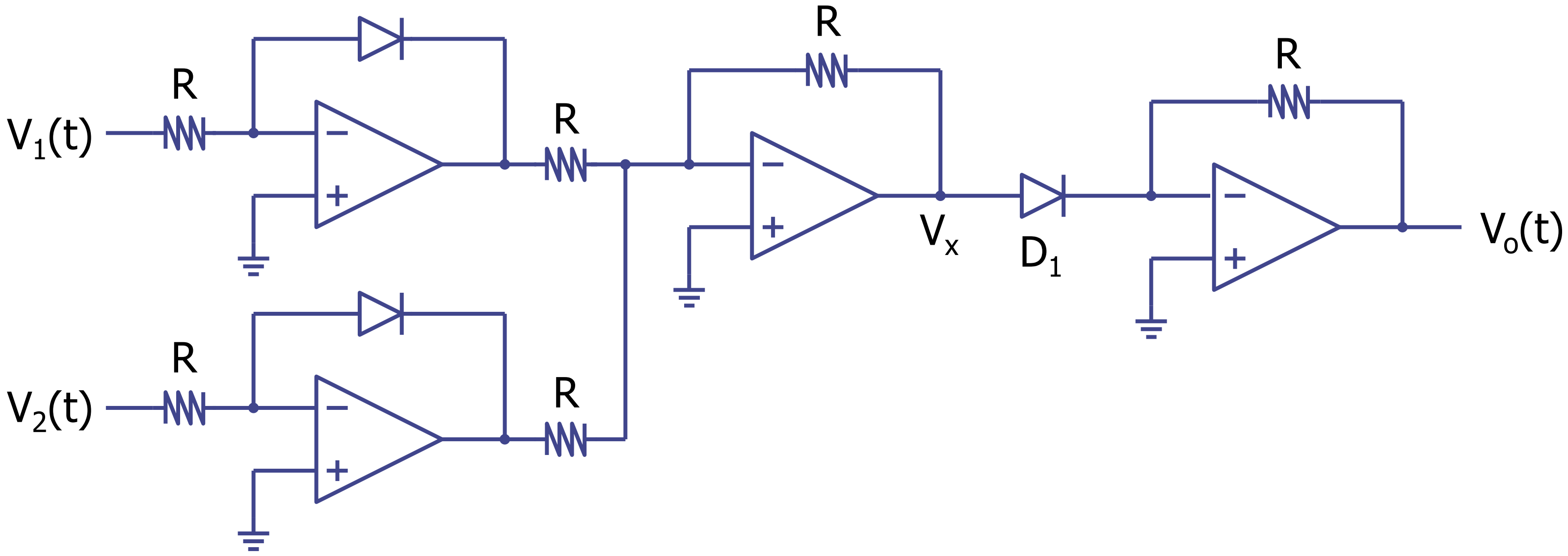


$$I(t) = I_S(e^{V_d(t)/V_T} - 1) \approx I_S e^{V_{in}(t)/V_T}$$

$$\Rightarrow \quad V_o(t) = -R_1 I(t) = -R_1 I_S e^{V_{in}(t)/V_T}$$

7.5.3 Multiplier Circuit

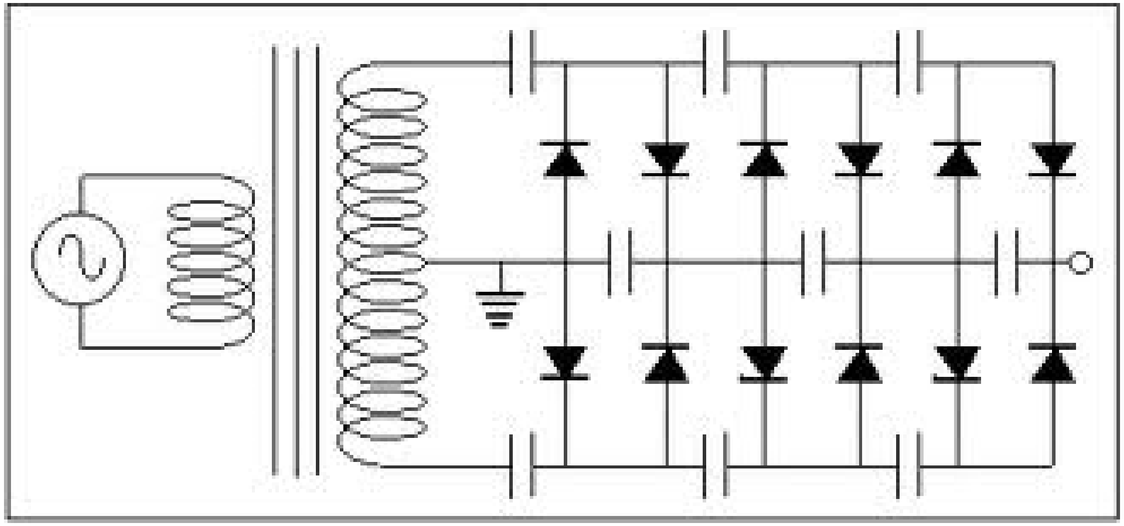
By combining the log and anti-log amplifiers, a Multiplier Circuit can be obtained.



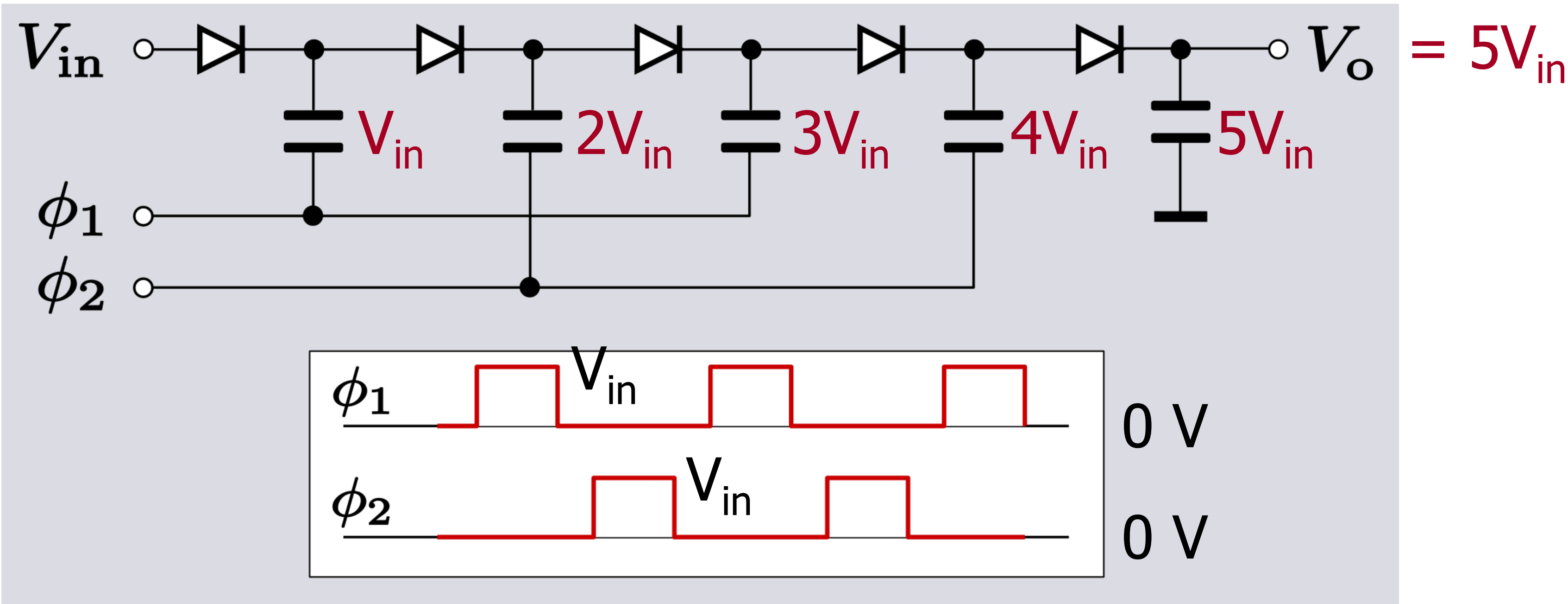
$$V_x \approx -\left[-V_T \ln\frac{V_1(t)}{RI_S} - V_T \ln\frac{V_2(t)}{RI_S}\right] = V_T \ln\frac{V_1(t)\cdot V_2(t)}{RI_S\cdot RI_S}$$

$$V_o(t) = -RI_S e^{V_x(t)/V_T} = -RI_S e^{\ln\frac{V_1(t)\cdot V_2(t)}{RI_S\cdot RI_S}} = -\frac{V_1(t)\cdot V_2(t)}{RI_S}$$

7.6 Complex Application Circuits



7.6.1 Charge Pump

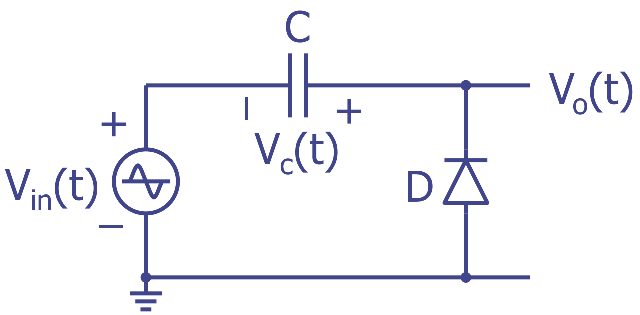


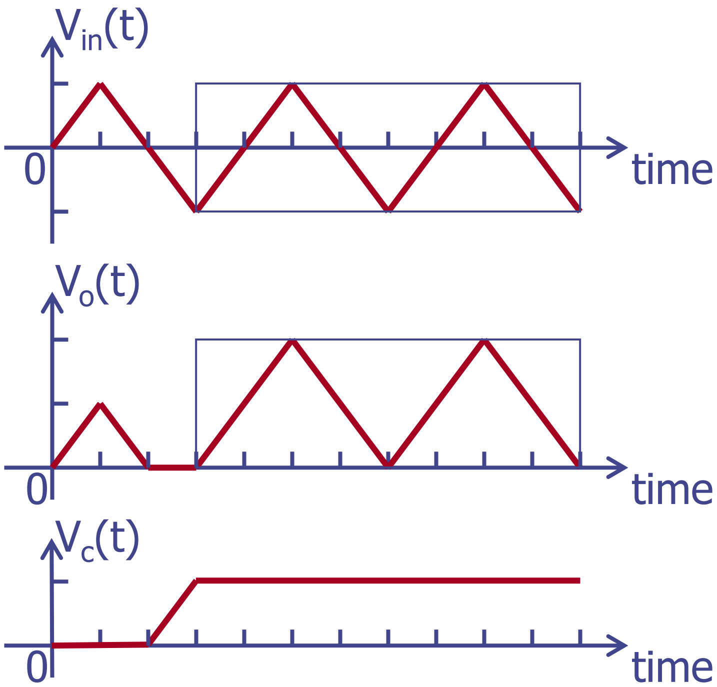
A charge pump is a DC-to-DC converter that takes a DC input voltage V_{in} and boosts it to a higher DC output voltage V_o. A two-phase clock of magnitude V_{in} is required.

7.6.2 Level Shift Circuit (Clamping Circuit)

A circuit with diode and capacitor can be used to bind the upper or lower extreme of a waveform to a fixed DC voltage level. These circuits are also known as DC voltage restorers.

Example 7-9: Sketch $V_o(t)$ assuming the capacitor is initially relaxed and the charging time of the capacitor is very small.

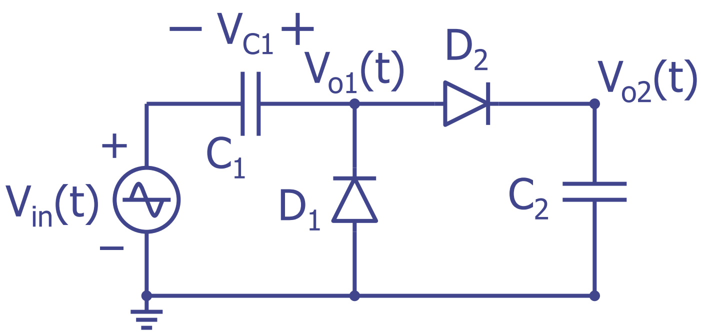


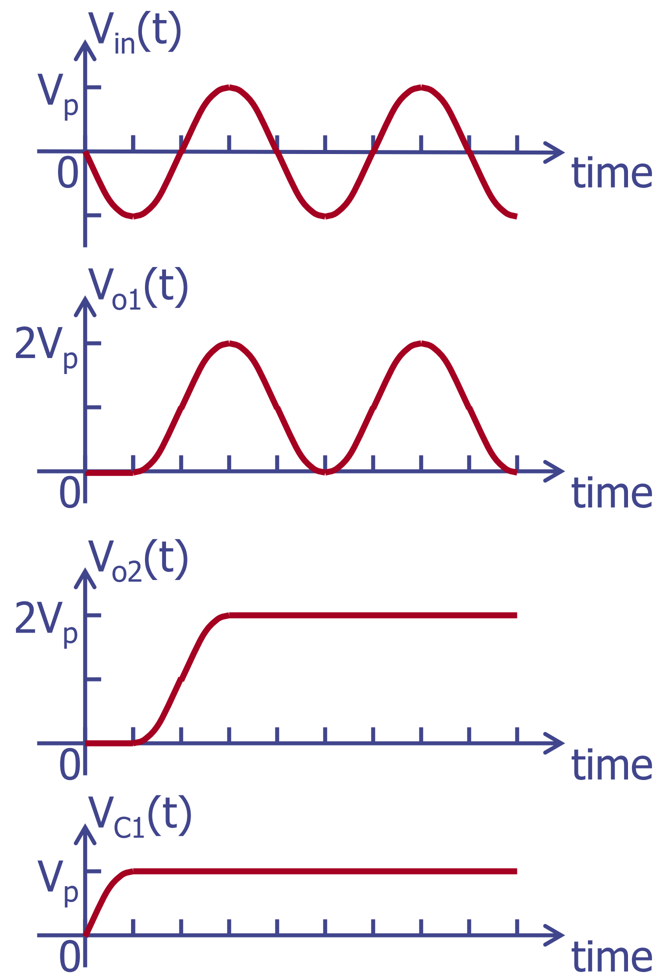


Once the capacitor is charged up by the most negative voltage, the rest of the signal is shifted up by that amount. The diode then becomes an open circuit.

7.6.3 Voltage Doubler and Multiplier

Example 7-10: Sketch $V_{o1}(t)$ and $V_{o2}(t)$ assuming the capacitors are initially relaxed and the charging time of the capacitors is very small.



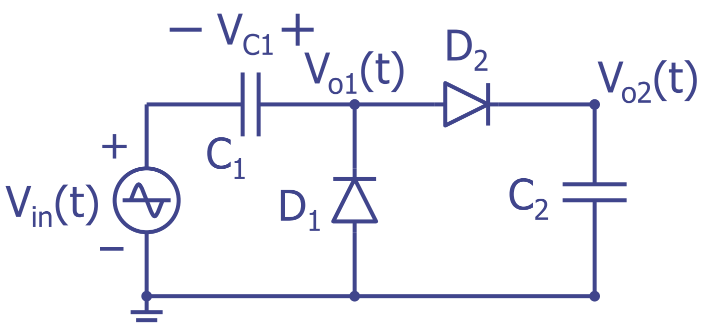


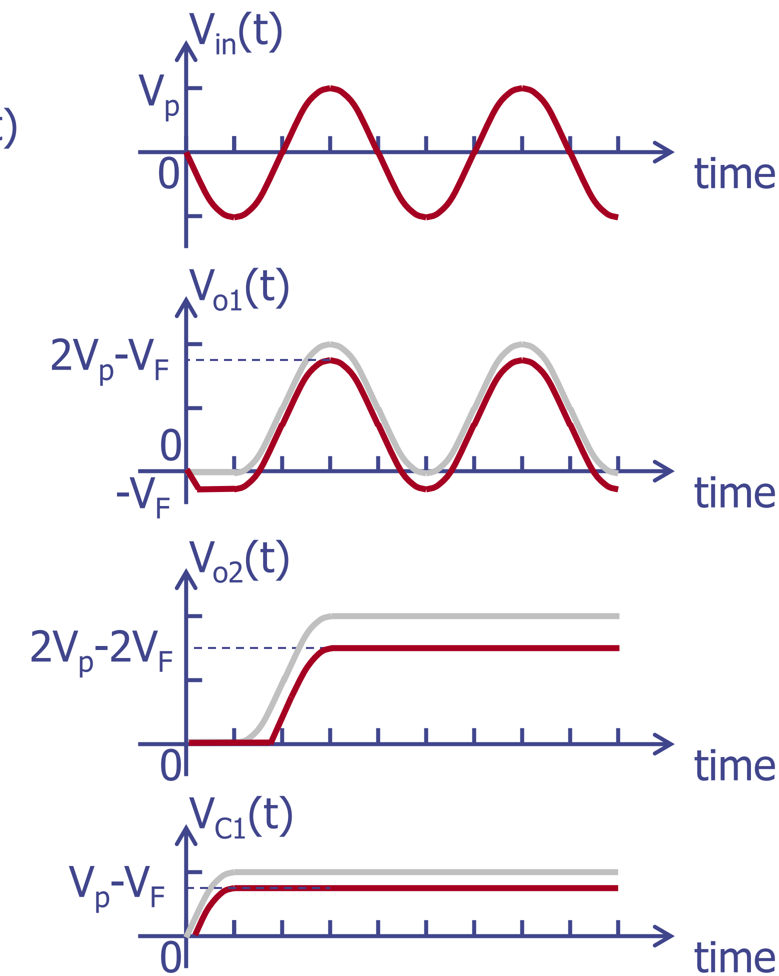
In practice, it will take a few cycles to fully charge C_1 and C_2.

The final output voltage is the peak-to-peak voltage of the input waveform.

Voltage Doubler Circuit (cont.)

Example 7-11: Sketch $V_{o1}(t)$ and $V_{o2}(t)$ using an offset diode model with V_F=0.7 V.

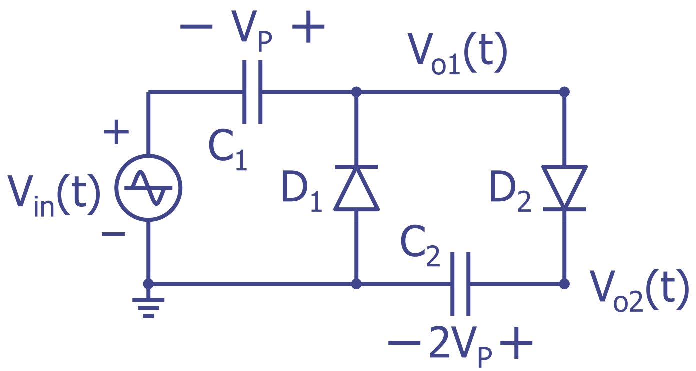




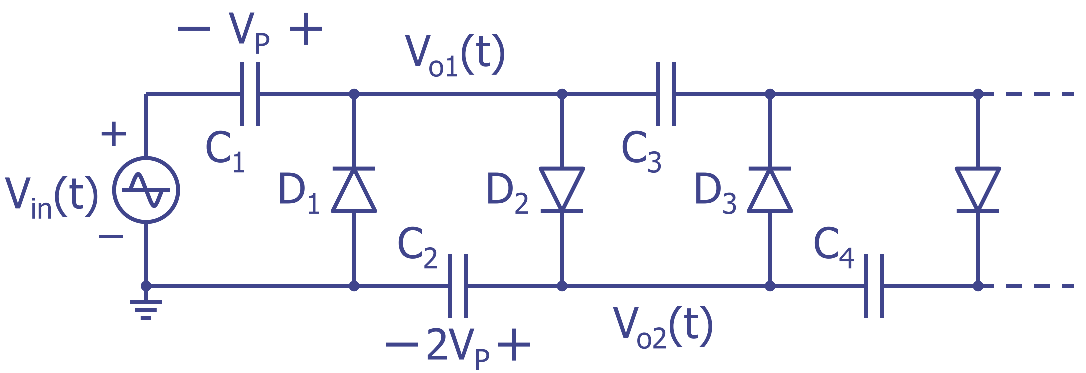
The final output voltage is $2V_p$-$2V_F$.

Voltage Multiplier Circuit

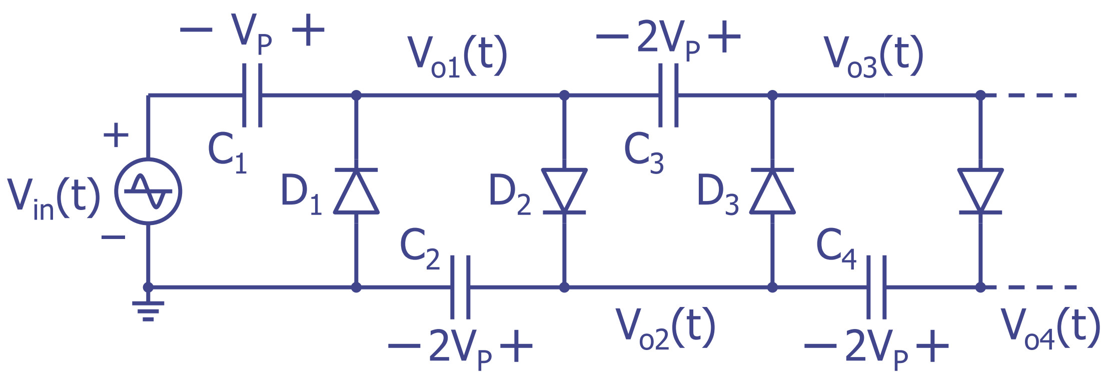
Redraw the previous voltage doubler circuit as:

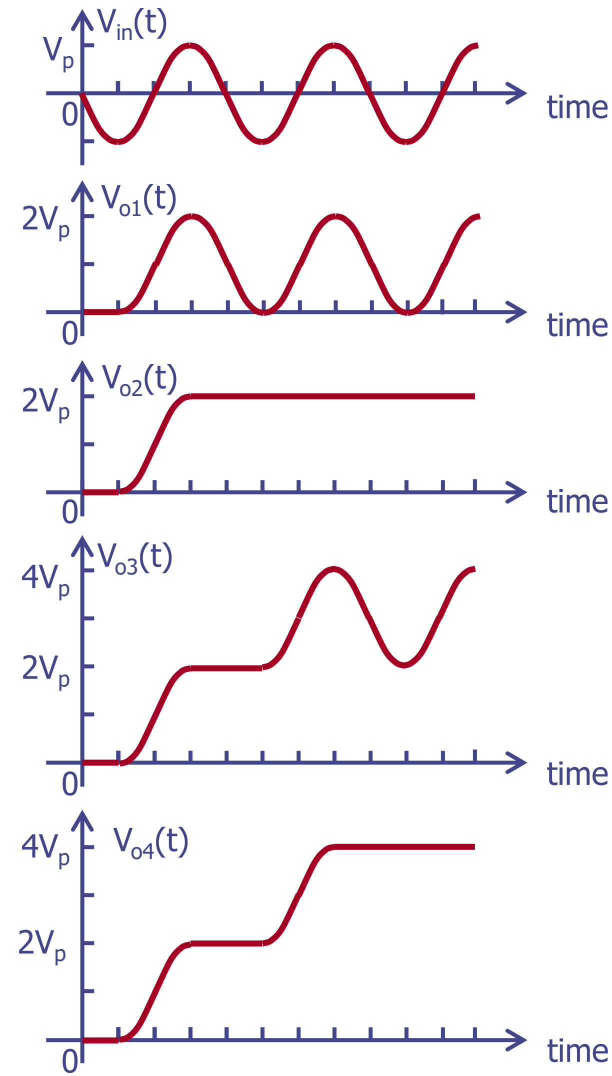


Add a few more stages:



Voltage Multiplier Circuit (cont.)





Each capacitor will be charged up to $2V_p$ after several cycles except C_1, which will be charged up to V_P.

Can provide DC voltage of $2V_p$, $4V_p$, ...