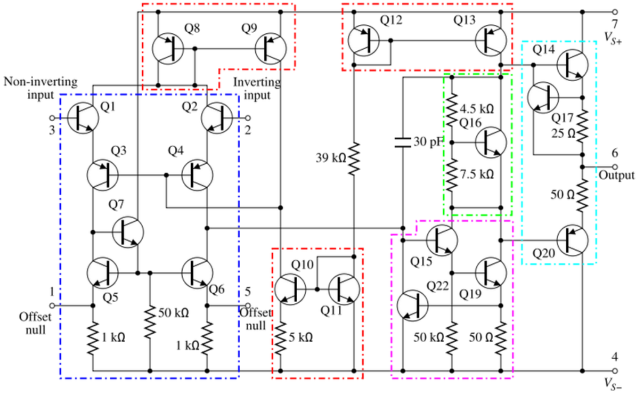
ELEC 2400 Electronic Circuits

Chapter 4: Op Amps and Circuits



µA741

Course Website: https://canvas.ust.hk

HKUST, 2021-22 Fall

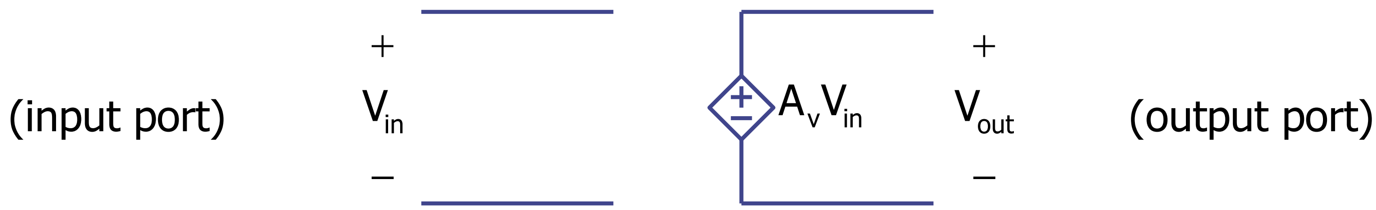
Chapter 4: Operational Amplifers and Circuits

4.1.1 Types of Amplifiers: Ideal Voltage Amplifier

An ideal voltage amplifier amplifies the input voltage signal V_{in} by a voltage gain A_v to generate the output voltage signal V_{out}:

$$V_{out} = A_v V_{in}$$

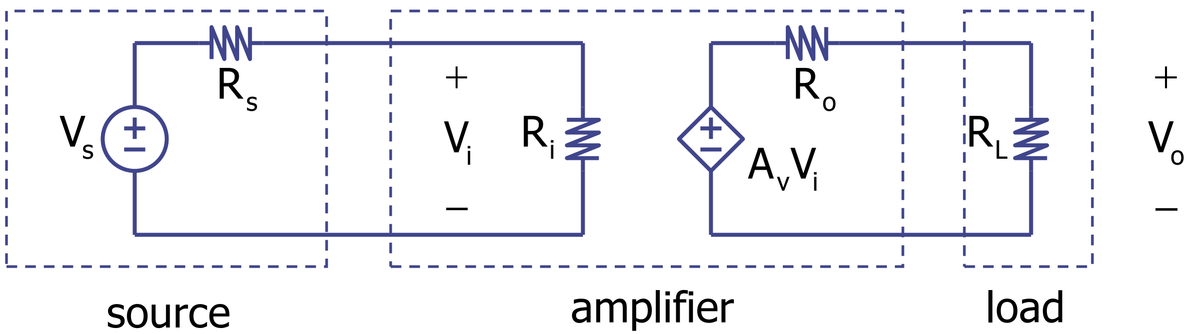
It can be modeled by a voltage-controlled voltage source:



Examples of voltage amplifiers:

Practical Voltage Amplifier

A practical voltage amplifier has finite input impedance R_i instead of infinity; and non-zero output impedance R_o instead of zero. The excitation - the source voltage, may also have finite source resistance R_s; and the response - the load, may be a load resistor R_L.

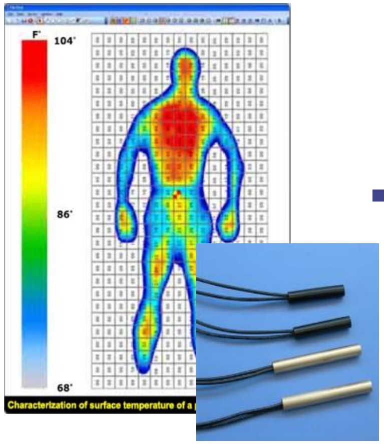


The effective voltage gain due to loading effects (R_s, R_i and R_o) is

$$\frac{V_{out}}{V_s} = \frac{R_i}{R_s + R_i}\frac{R_L}{R_o + R_L}A_v$$

Example 4-1

Example 4-1: Compute the voltage gain of the voltage amplifier needed for the temperature sensor below, if we require to have V_o = 1 V when V_s = 100 mV (at 100 ^oC).



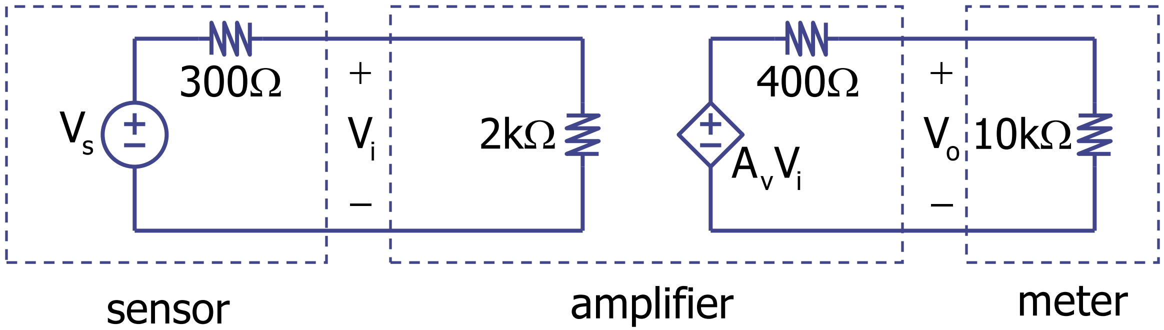
0 °C 100 °C

0 V 1 V

Temperature Sensor	$+$ V_i $-$	Voltage Amplifier	$+$ V_o $-$	Meter Display
V_s = 0 V at 0 ^oC +1 mV/^oC R_s = 300 Ω		R_i = 2 kΩ R_o = 400 Ω		R_L = 10 kΩ

Example 4-1 (cont.)

Soln.: The temperature sensor can be modeled as follows:



$$V_{out} = A_v V_i \frac{10k}{400+10k}$$

$$= A_v V_s \frac{2k}{300+2k} \frac{10k}{400+10k}$$

$$\Rightarrow \quad 1V = A_v \times 100mV \frac{2k}{300+2k} \frac{10k}{400+10k}$$

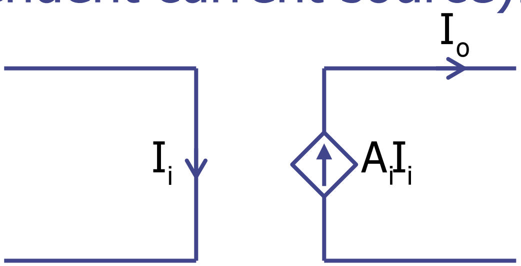
$$\Rightarrow \quad A_v = 11.96 \text{ V/V} = 21.6\text{dB}$$

$$\boxed{\text{dB} = 20\log_{10}(\text{gain})}$$

Current Amplifier

A current amplifier can be modeled as a current-controlled current source, with a current gain of A_i. The ideal current amplifier has zero input impedance (R_i=0) and infinite output impedance (R_o=∞, in parallel with the dependent current source).

$$\frac{I_o}{I_i} = A_i$$



Examples of current amplifiers:

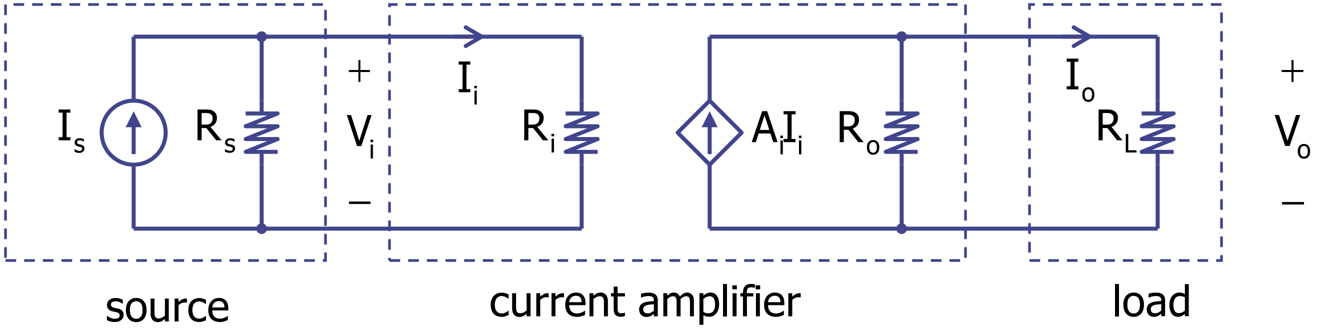
Current Source

Hi Fi Power Amplifier

Speaker

Practical Current Amplifier

A practical current amplifier has non-zero (instead of zero) input impedance R_i and finite (non-infinite) output impedance R_o.



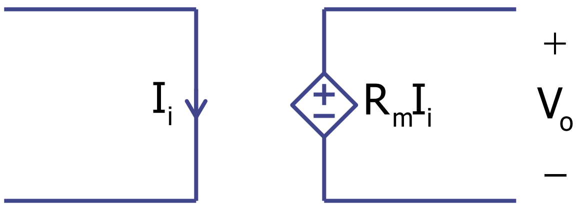
The effective current gain due to loading effects (R_s, R_i and R_o) is

$$\frac{I_o}{I_s} = \frac{R_s}{R_s + R_i} \frac{R_o}{R_o + R_L} A_i$$

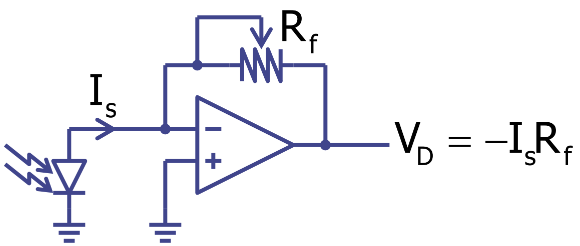
Transresistance Amplifier

A transresistance amplifier can be modeled as a current-controlled voltage source, and the gain R_m has the dimension of resistance. The ideal transresistance amplifier has zero input impedance (R_i=0) and zero output impedance (R_o=0).

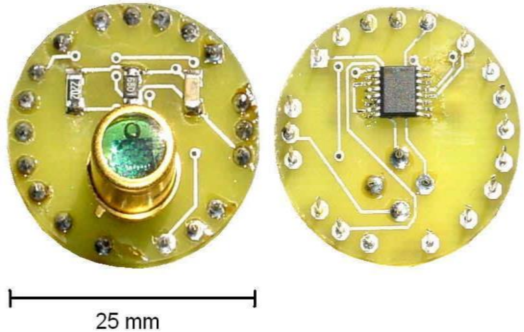
$$\frac{V_o}{I_i} = R_m$$



Examples of transresistance amplifiers:



Photodiode Amplifier

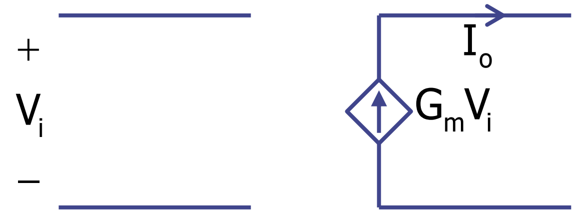


Optical Sense Amplifier

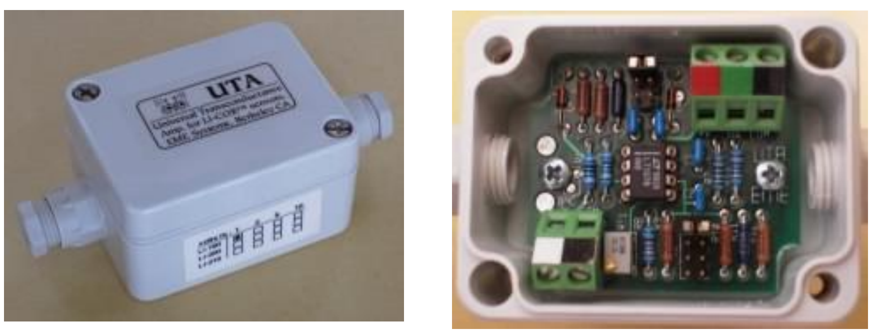
Transconductance Amplifier

A transconductance amplifier can be modeled as a voltage-controlled current source, and the gain G_m has the dimension of conductance. The ideal transconductance amplifier has infinite input impedance ($R_i=\infty$) and infinite output impedance ($R_o=\infty$).

$$\frac{I_o}{V_i} = G_m$$



Examples of transconductance amplifiers:



Universal Transconductance Amplifier

Fluke Transconductance Amplifier

4.1.2 Operational Amplifier (Op Amp)

Operational amplifiers, or commonly known as op amps, are probably the most versatile electronic components used in analog circuits. The first monolithic IC (integrated circuit) implementation of an op amp, labeled μA702, was designed by Bob Widlar of Fairchild Semiconductor in 1963.

Op amps are so named because they can be configured to perform mathematical operations such as

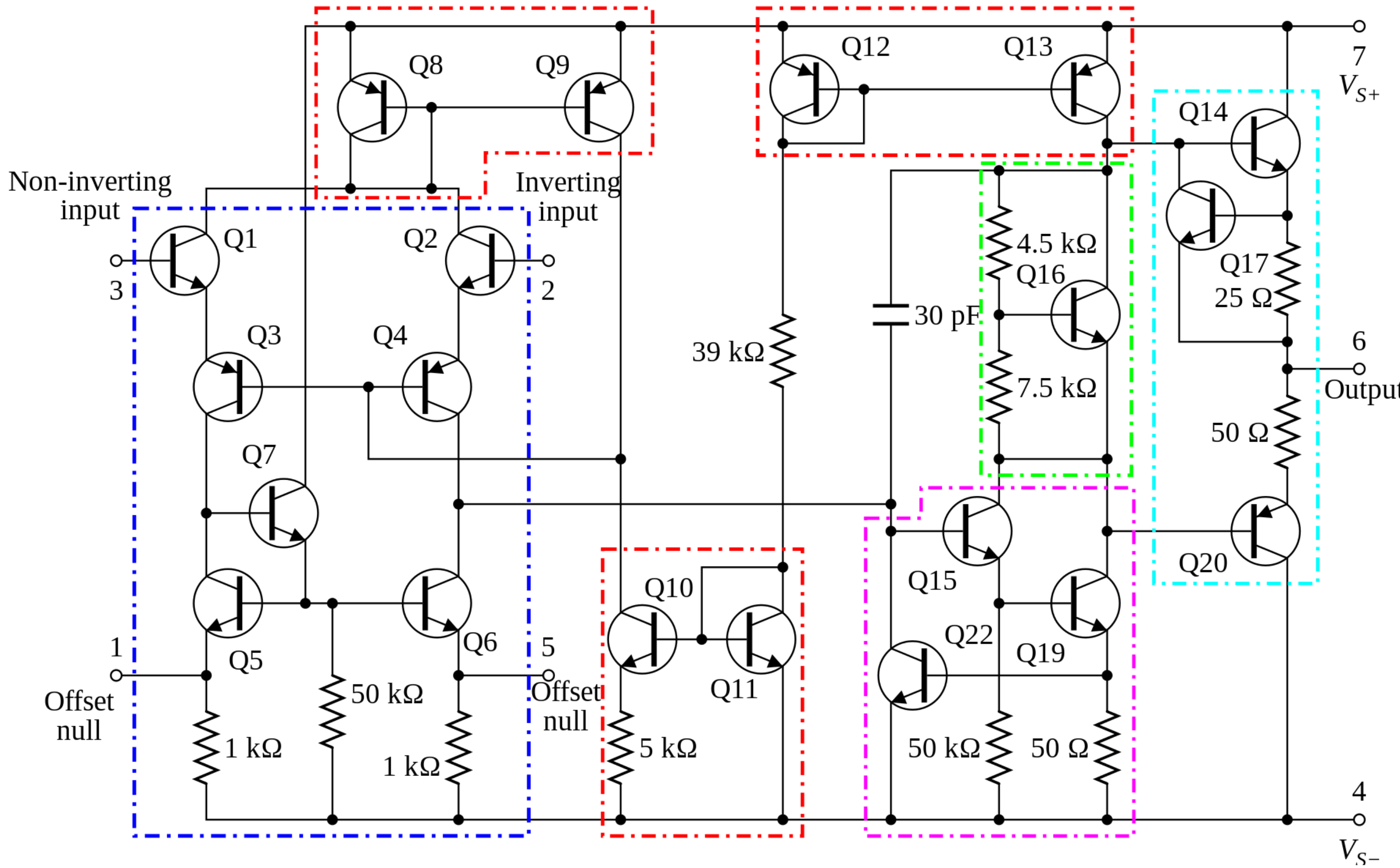
$$+,\ -,\ \times,\ \div,\ \int,\ d/dx,\ \log,\ \exp,\ \text{etc.}$$

Besides they can perform many signal processing functions like amplification, filtering, A/D and D/A conversions, etc.

Robert Widlar
1937 – 1991

"At one point in time, Widlar designed more than 80% of the linear ICs made and sold in the world".
https://en.wikipedia.org/wiki/Bob_Widlar

The Most Successful Op Amp: μA741

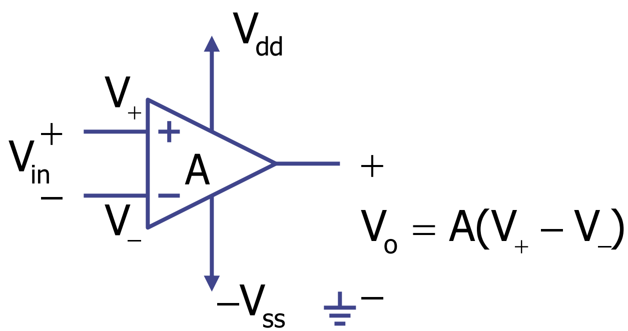


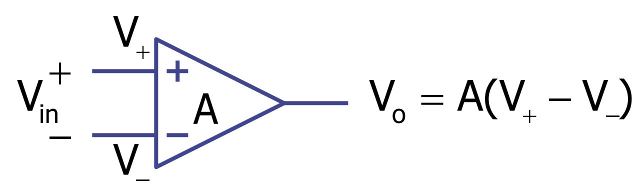
Current mirrors, Differential amplifier, Class A gain stage, Voltage level shifter, Output stage.

Designed by Dave Fullagar of Fairchild Semiconductor in 1968.
https://en.wikipedia.org/wiki/Operational_amplifier

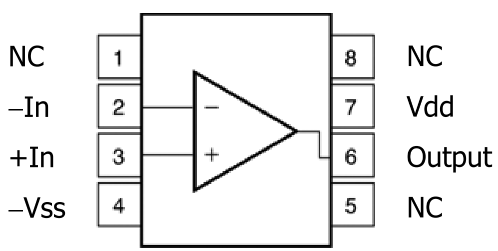
Op Amp as High Gain Voltage Amplifier

An op amp is a voltage amplifier with an extremely high voltage gain (for example, A_v > 10000 V/V, or 80 dB), while the ideal op amp has infinite gain.





Examples of op amp (μA741):



Ideal and Practical Op Amps

The op amp can be described as a differential input single-ended output device.

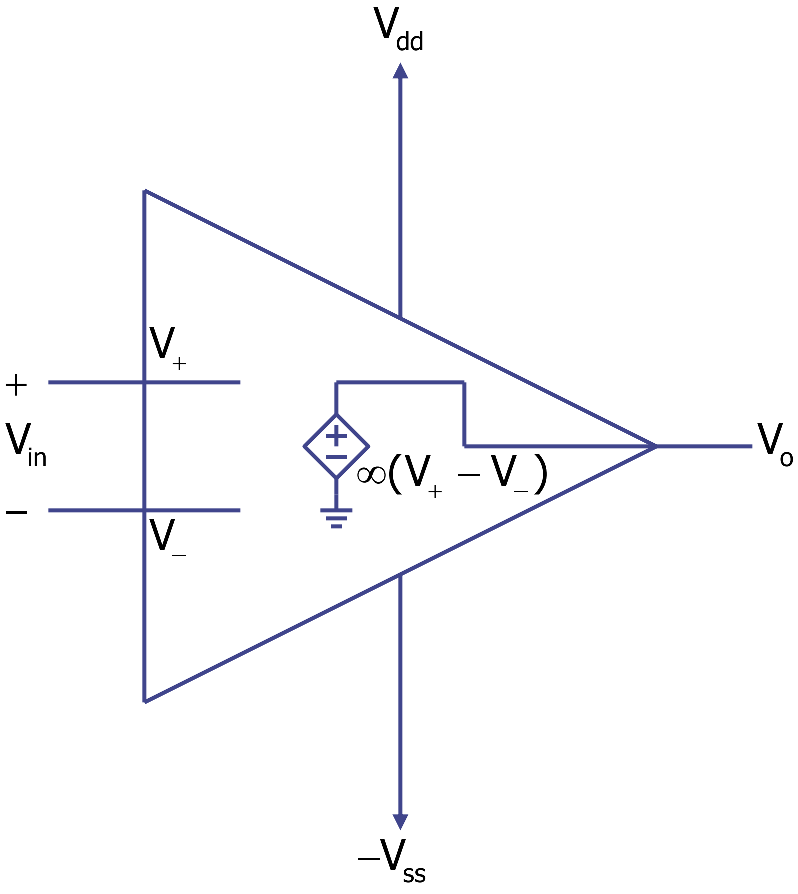
In the past, positive and negative power supplies were used, such as ±15 V or ±5 V. Nowadays, the negative power supply is often just 0 V (Ground, or GND), and the zero output can be defined anywhere between GND and positive supply.

Some of the properties of an op amp are shown below:

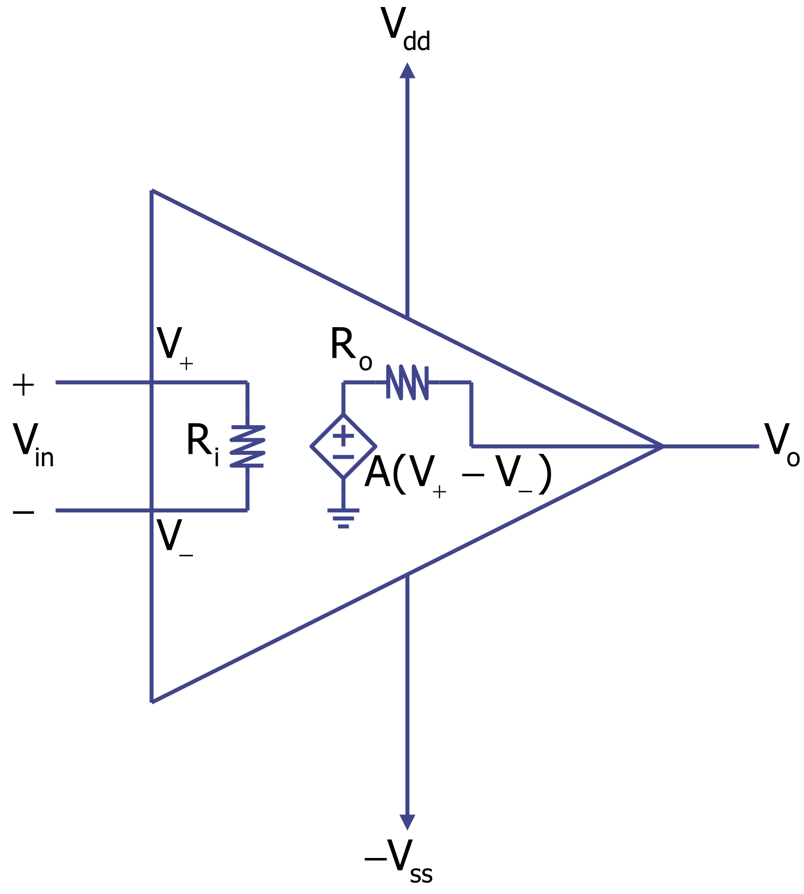
	Ideal	Practical
Gain	$A = \infty$	10^5 V/V $\rightarrow$ 100 dB
Input Impedance	$R_i = \infty$	10 MΩ
Output Impedance	$R_o = 0\ \Omega$	100 Ω
Input offset voltage	$V_{os} = 0$ V	±2 mV
Input offset current	$I_+ - I_- = 0$ nA	±5 nA
Bandwidth	$BW = \infty$	100 MHz (using feedback)

Circuit Model of Op Amp

Ideal Op Amp

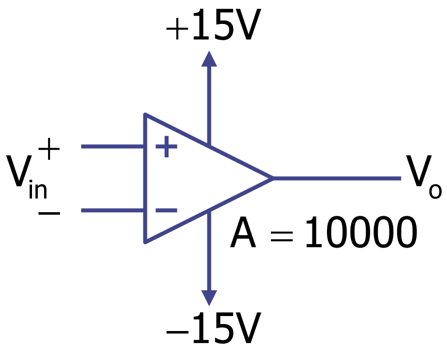


Practical Op Amp



Example 4-2

Example 4-2: Compute the output voltage V_o when V_{in} is (1) 0 V; (2) 100 μV; (3) 1 mV; and (4) 10 mV.



Soln.:

(1) $V_o = 10000 \times 0 = 0$ V

(2) $V_o = 10000 \times 100\ \mu = 1$ V

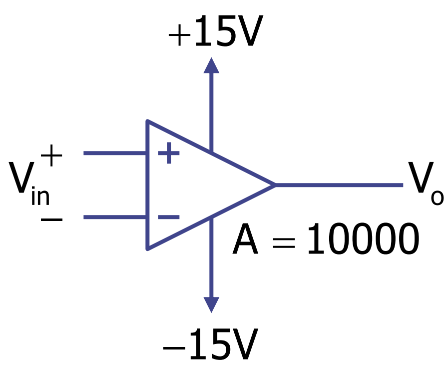
(3) $V_o = 10000 \times 1\ \text{m} = 10$ V

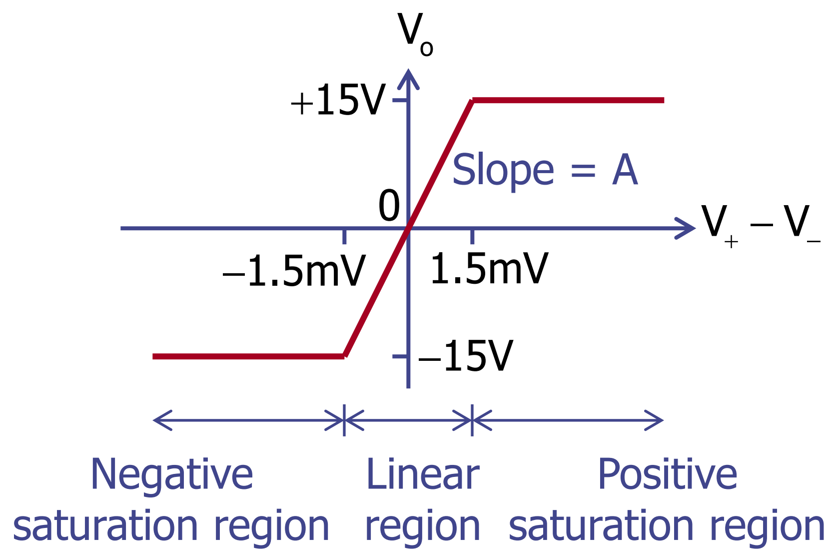
(4) $V_o = 10000 \times 10\ \text{m} = 100$ V

However, 100 V is even higher than the power supply voltage +15 V, and the output voltage will saturate at +15 V.

Operating Regions of Op Amp

If the input voltage is too large, the output voltage will saturate at either the positive power rail (with $V_+ - V_-$ too positive) or the negative power rail (with $V_+ - V_-$ too negative). For example, consider the op amp shown below.





The input voltage $V_{in} = V_+ - V_-$ can only range between −1.5 mV and +1.5 mV and is too narrow to be practical. Therefore, op amps are often used in a feedback configuration.

Why Infinite Gain for Op Amp?

Because we can’t do better than that!
Integrated circuit (IC) technology, despite its many advantages, comes with a general lack of precision circuit elements, i.e., no precision values for resistance, capacitance, gain, etc., despite good matching of devices within the same die. Many parameters come with huge temperature coefficients. There are also batch-to-batch variations.

Bottom line, having a precision gain is both difficult and costly in IC. An op amp having a fixed gain is also not flexible.

So rather than dealing with a finite but inaccurate gain
We prefer a very high (but inaccurate) gain and employ feedback techniques to overcome the shortcomings.

Many times, the passives, i.e., resistors, capacitors, are more economical to stay external than to be integrated on the IC. You can then make them as accurate as you want.

The op amp ends up much more versatile this way!!!

Feedback

Terminology

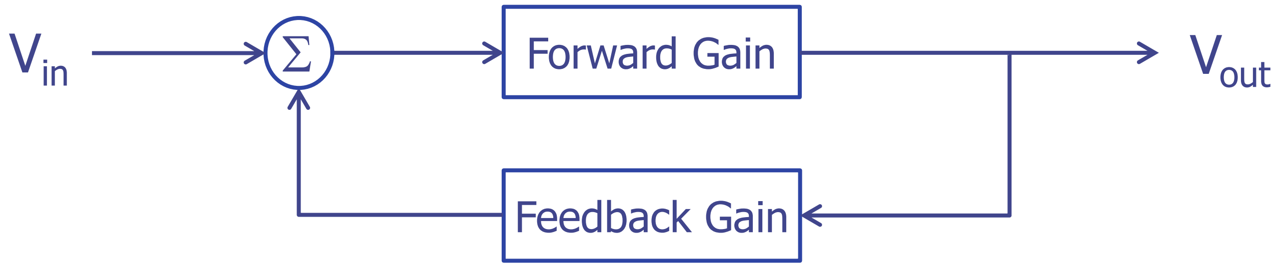
Feedback: Some of the output is routed back to and combined with the input.

Positive Feedback: The sign of the feedback is such that it tends to reinforce the output until it grows to a maximum.

Negative Feedback: The sign of the feedback is such that it tends to stabilize the output toward a target goal.

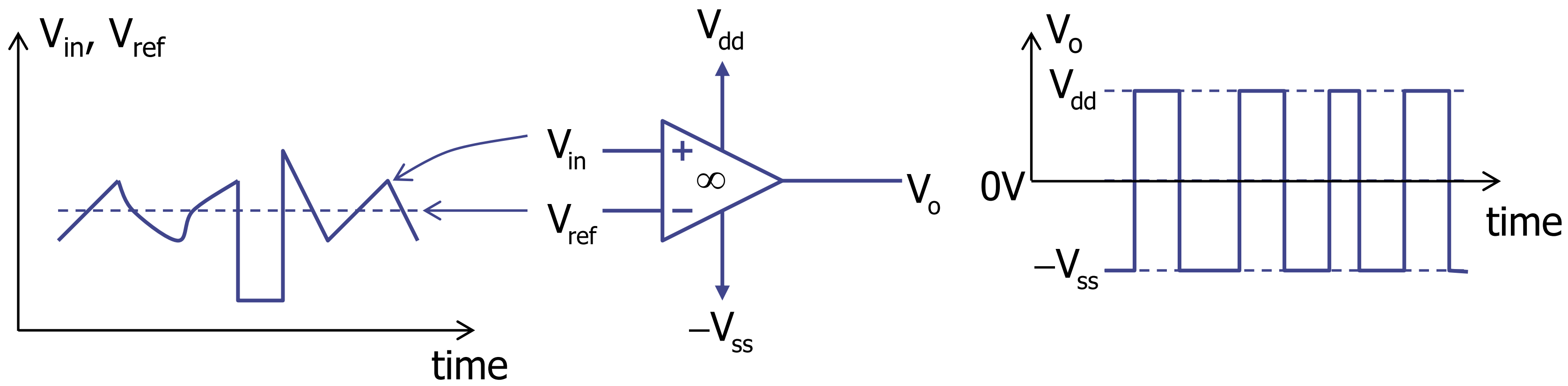
Open-Loop: No feedback.

Closed-loop: With feedback.



Op Amp as Comparator

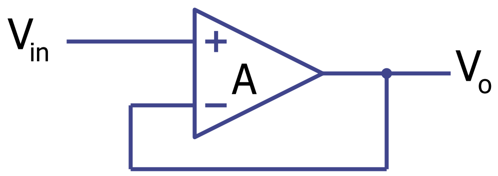
Due to the high gain of the op amp, it can be used as a comparator: when the input voltage V_{in} is higher than the reference voltage V_{ref}, the output voltage V_o is at V_{dd}; and when $V_{in} < V_{ref}$, $V_o = -V_{ss}$.

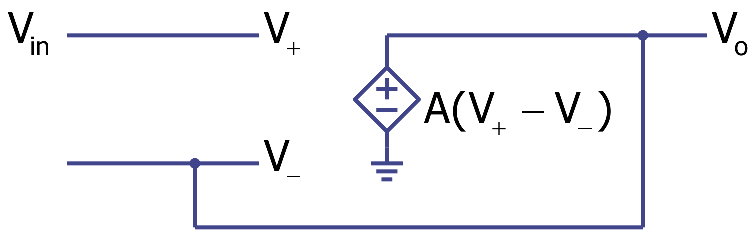


It should be noted that although feasible, it is not economical to use an op amp as a comparator.

4.1.3 Unity Gain Buffer

The simplest feedback configuration is the unity gain buffer, by connecting the output voltage V_o back to the negative input terminal V_- as shown below. The connection realizes negative feedback.





The output voltage V_o can easily be computed:

$$V_o = A(V_{in} - V_o)$$

$$V_o = \frac{A}{1+A} V_{in}$$

$$\approx V_{in} \quad (A >> 1)$$

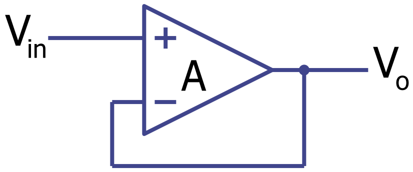
4.1.4 Positive Feedback vs Negative Feedback

Qn. What is the difference between positive feedback and negative feedback?

Ans. Negative feedback is stable and positive feedback is unstable (loosely speaking).

Qn. Why?

Negative Feedback

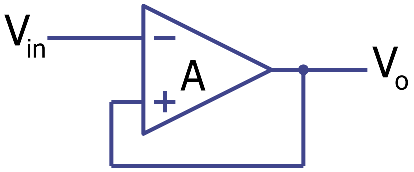


$$V_o = A(V_{in} - V_o)$$

$$\frac{V_o}{V_{in}} = \frac{A}{1 + A}$$

$$\approx 1 \ (A >> 1)$$

Positive Feedback



$$V_o = A(V_o - V_{in})$$

$$\frac{V_o}{V_{in}} = \frac{-A}{1 - A} = \frac{A}{A - 1}$$

$$\approx 1 \ (A >> 1)$$

Problem with Positive Feedback

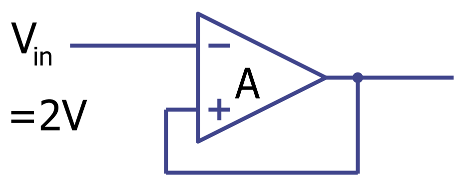
For the positive feedback connection, let A=1001. For V_{in}=2 V, then V_o=2.002 V.

Next, consider injection of noise at $V_o(0)$ such that it drops slightly to 2.001 V. Recompute V_o and $V_o(1^{st})$ = 1.001 V.

Go through the loop one more time and $V_o(2^{nd})$ = –1000 V!

Clearly this is unrealistic, and V_o will stay happily at the negative power rail! Similarly, if a disturbance causes V_o to increase slightly from 2.002 V, it will go all the way to the positive power rail.

Unstable



$$V_o = \frac{A}{A-1} V_{in}$$

0. 2.002V

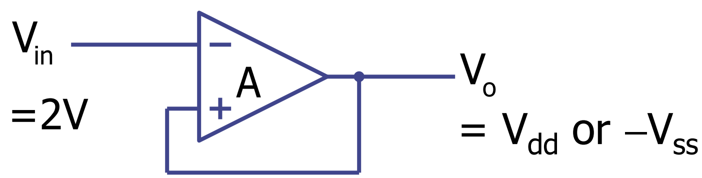
noise

2.001V

1. 1.001V

2. –1000V

Final State (Railed at Power Supply)



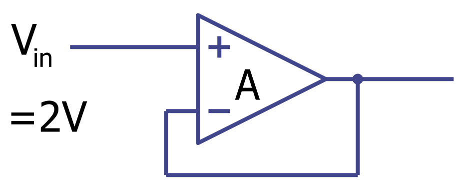
Negative Feedback Provides Stability

For the negative feedback connection, let A=999. For V_{in}=2 V, then V_o=1.998 V.

Next, consider injection of noise at $V_o(0)$ such that it drops slightly to 1.997 V. Recompute V_o and $V_o(1^{st})$ = 2.997 V.

However, as V_o swings from 1.997 V to 2.997 V, it has to pass through 1.998 V, which is the solution to the circuit. Hence, when V_o reaches 1.998 V it will just stay there! Hence negative feedback provides stability.

Stable



$$V_o = \frac{A}{A+1} V_{in}$$

1. 2.997V

0. 1.998V

noise

1.997V

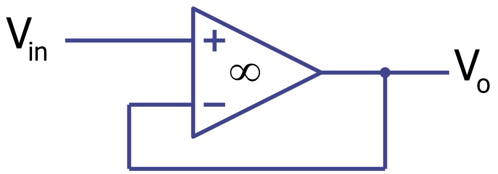
In summary, given a deviation from the operating point:
Positive Feedback - it will deviate further out ⇒ Instability
Negative Feedback - it will bring it back ⇒ Stability

Unity Gain Buffer with Ideal Op Amp

For the following op amp circuits, we will consider ideal op amps, that is, $A = \infty$, $R_i = \infty$, and $R_o = 0\ \Omega$.

Reconsider the unity gain buffer. If $A = \infty$, then

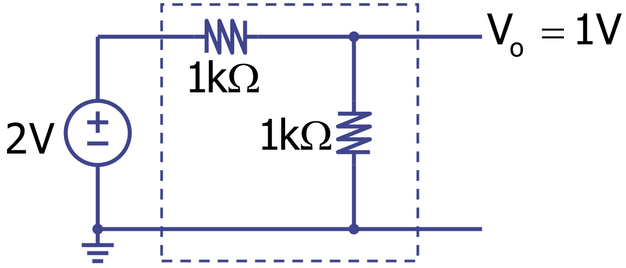
$$\begin{aligned} V_o &= \frac{A}{1+A} V_{in} \\ &= \frac{\infty}{1+\infty} V_{in} \\ &= V_{in} \\ \Rightarrow \qquad V_+ &= V_- \end{aligned}$$



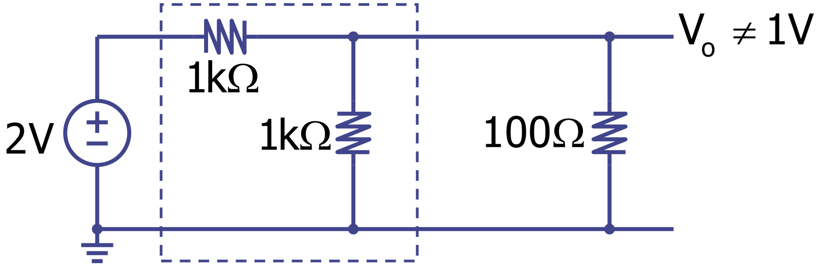
N.B. When an ideal op amp is operating in linear region (not in positive/negative saturation region) and under proper negative feedback conditions, the positive input terminal V_+ has the same potential as the negative input terminal V_-.

Unity Gain Buffer can drive Resistive Load

A unity gain buffer is different from just a simple wire connected between the input and output of the buffer because the output of a unity gain buffer can drive resistive load.

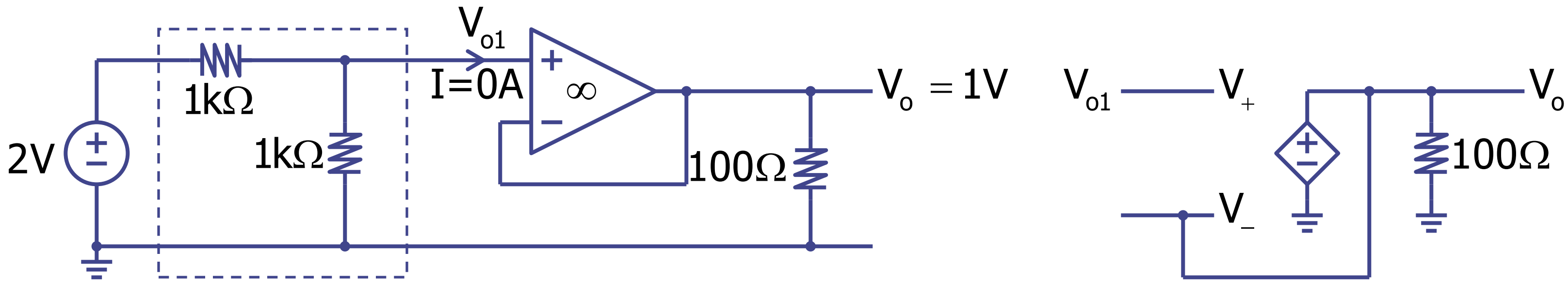


Resistive divider



Divider cannot drive resistor

The "internal" controlled voltage source enables the op amp to drive a resistive load.



Example 4-3

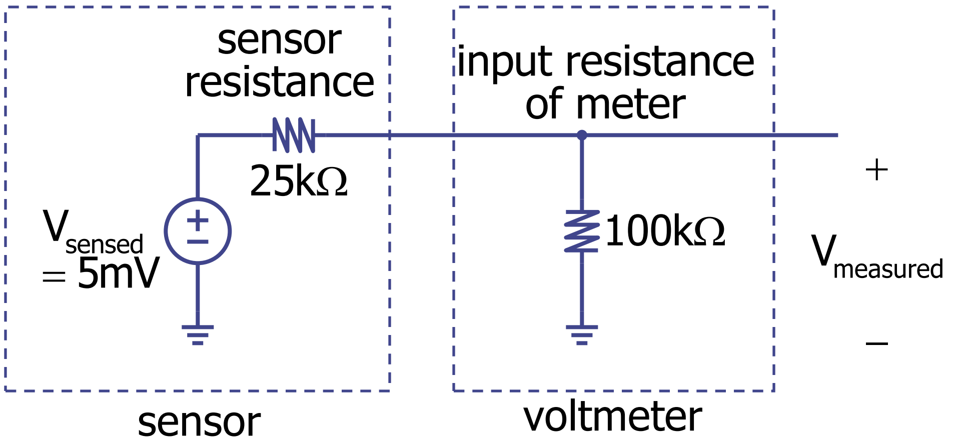
Example 4-3: A sensor outputs a sensed voltage of 5 mV and has an internal resistance of 25 kΩ. (1) If the sensor is connected to a voltmeter with input resistance of 100 kΩ, what is the measured voltage? (2) When a unity gain buffer is added between the sensor and the voltmeter, what is the measured voltage?

Soln.:

(1)

$$V_{measured} = 5m \times \frac{100k}{25k + 100k}$$

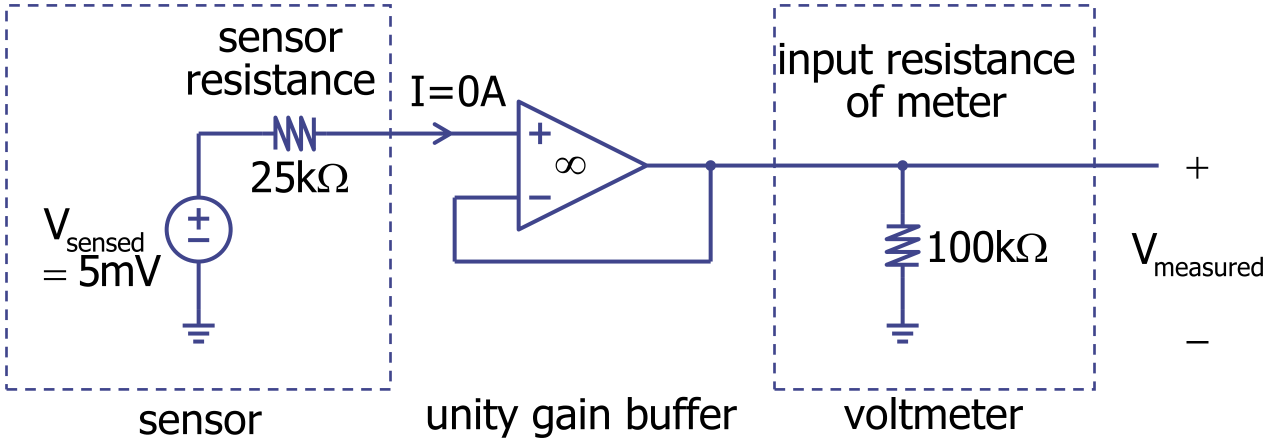
$$= 4mV$$



The measured value is only 4 mV and is off (lowered) by 20%.

Example 4-3 (cont.)

(2)



Since $R_i = \infty$, I = 0 A; therefore,

$$V_+ = V_{sensed} = 5\text{mV}$$

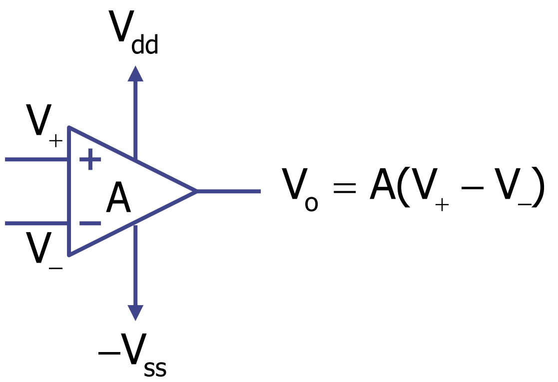
$$\Rightarrow \quad V_{measured} = V_- = V_+ = V_{sensed} = 5\text{mV}$$

Using a unity gain buffer gives an accurate measured valued of the sensor output voltage.

4.1.5 Ideal Op Amp Characteristics

In order to analyze more complicated op amp circuits, we need to know two important characteristics of the ideal op amp.

(1) $V_+ = V_-$



If the op amp operates in linear region (not in positive/negative saturation region) with negative feedback connection, the output is a finite voltage.

Since $$V_+ - V_- = \frac{V_o}{A}$$

for an ideal op amp, $A = \infty$, then

$$V_+ - V_- = \frac{\text{finite number}}{\infty} = 0$$

$$\Rightarrow \quad V_+ = V_-$$

For this to be true, $-V_{ss} < V_+, V_-, V_o < V_{dd}$. This condition must be checked every time!!!

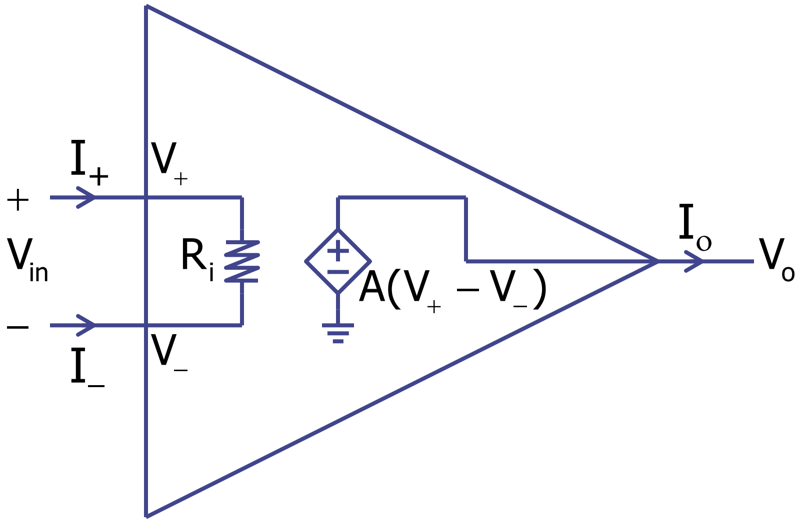
Ideal Op Amp Circuits (cont.)

(2) $I_+ = I_- = 0$

The input currents of an op amp depend on the input resistance.

For an ideal op amp, $R_i = \infty$.

$$I_+ = -I_- = \frac{V_{in}}{R_i} = \frac{V_{in}}{\infty} = 0$$

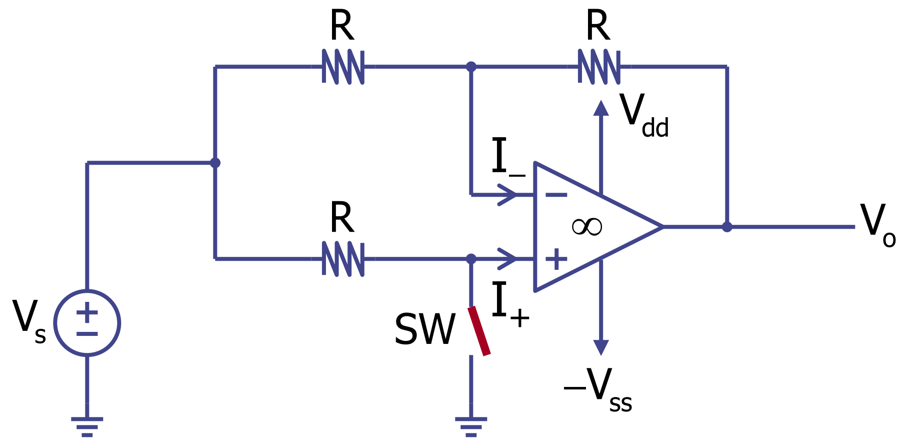


Therefore, the input currents of an ideal op amp are always zero.

N.B. I_o is not necessarily equal to 0.

Example 4-4

Example 4-4: Find V_o when the switch is (i) open; and (ii) closed.



Soln.:

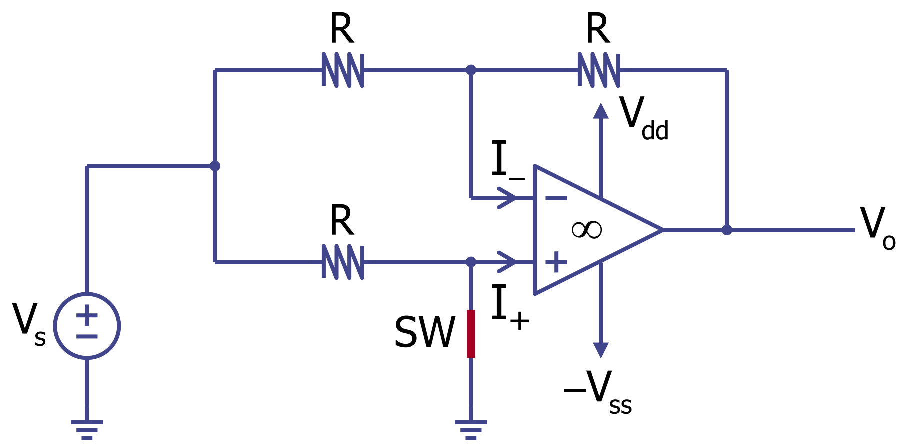
(i) When the switch is open and since $I_+ = 0$, the voltage drop across R that is connected to V_+ is 0, giving

$$V_s = V_+ \left(= V_-\right)$$

and $I_- = 0$ gives $$\frac{V_s - V_-}{R} = \frac{V_- - V_o}{R} \Rightarrow \frac{V_s - V_s}{R} = \frac{V_s - V_o}{R}$$

$$\Rightarrow \quad V_o = V_s$$ (a voltage follower circuit)

Example 4-4 (cont.)



(ii) When the switch is closed, V_+ is shorted to ground, and

$$V_+ = 0 \left(= V_-\right)$$

Now $I_- = 0$,

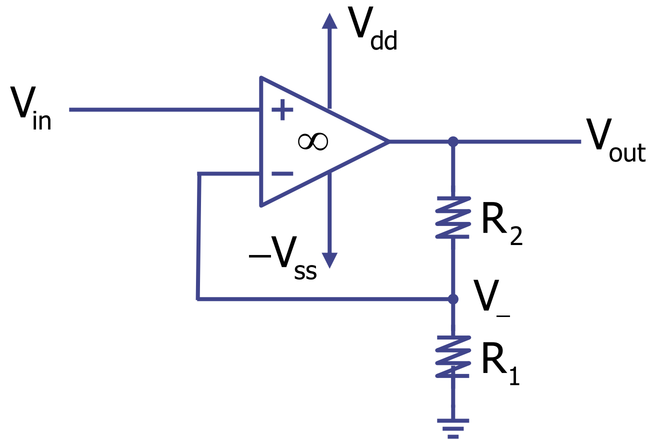
$$\frac{V_s - 0}{R} = \frac{0 - V_o}{R}$$

$\Rightarrow \quad V_o = -V_s$ (a voltage inverter circuit)

Chapter 4: Operational Amplifers and Circuits

4.2.1 Non-Inverting Amplifier

To achieve a gain other than unity, we have two simple configurations: the non-inverting amplifier and the inverting amplifier. The non-inverting amplifier is shown below.



Now,

$$V_{in} = V_{+} = V_{-}$$

and

$$V_{-} = \frac{R_1}{R_1 + R_2} V_{out} = V_{in}$$

$$\Rightarrow \quad V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$

The gain of the amplifier is determined by two external resistors, which can be precisely chosen, and pretty much independent of the imperfections in the op amp. This is wonderful!

Op Amp Gain and Closed Loop Gain

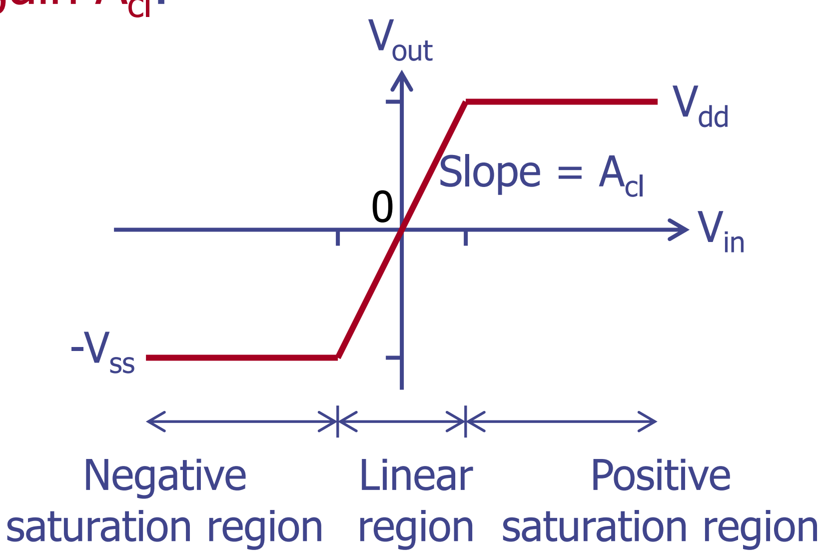
It is important to distinguish two types of voltage gains.
(1) The op amp gain A has the relation:

$$V_{out} = A(V_+ - V_-)$$

In the previous case, $A = \infty$.

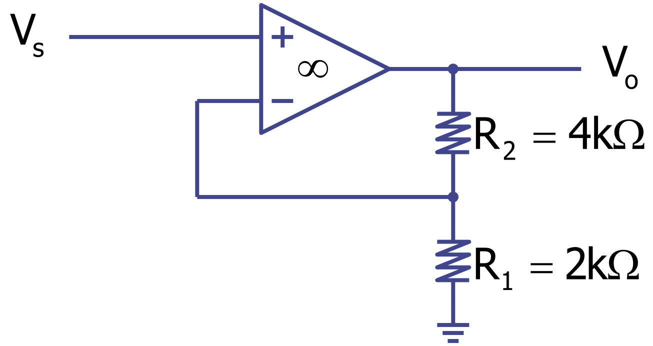
(2) The non-inverting amplifier employs negative feedback, and the voltage gain relating the output voltage to the input voltage is the closed loop gain A_{cl}:

$$A_{cl} = \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}$$



Example 4-5

Example 4-5: Let the op amp be ideal, (i) find V_o for V_s = 2 V; and (ii) find $V_o(t)$ for $V_s(t) = 2\sin(2\pi 50t)$ V.

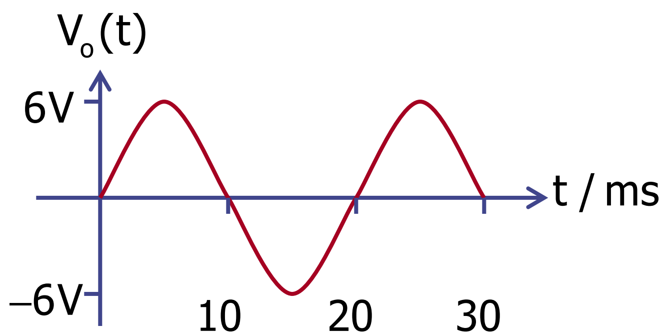


Soln.:

(i) $$\frac{V_o}{V_s} = 1 + \frac{4k}{2k} = 3$$

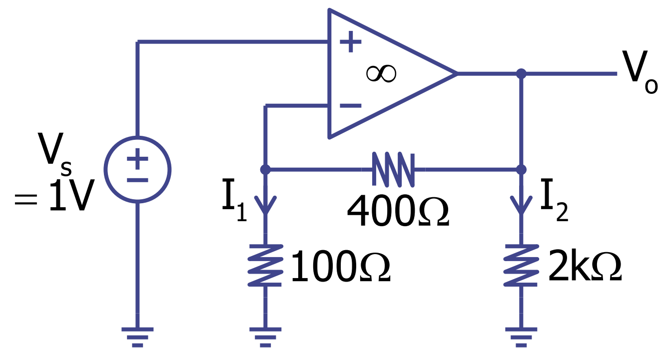
$$\Rightarrow \quad V_o = 2 \times 3 = 6V$$

(ii) $$V_o(t) = 6\sin(2\pi 50t)\ V$$



Example 4-6

Example 4-6: Find I_1 and I_2.



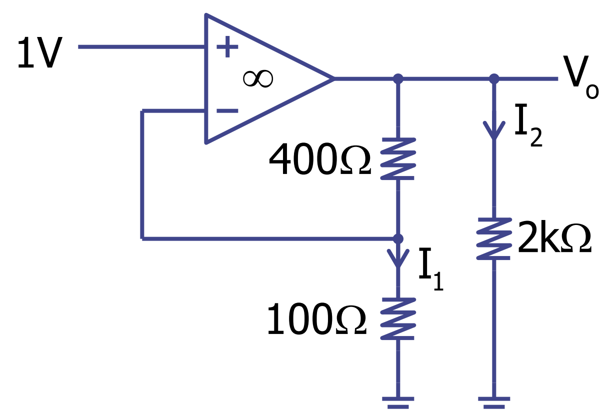
Soln.:

The op amp circuit can be redrawn as shown in the right.

$$I_1 = \frac{V_-}{100} = \frac{V_+}{100} = \frac{1}{100} = 10mA$$

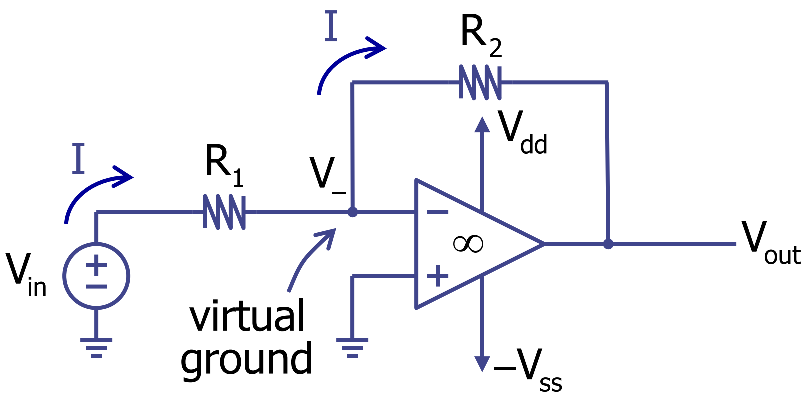
$$V_o = \left(1 + \frac{400}{100}\right) \times 1V = 5V$$

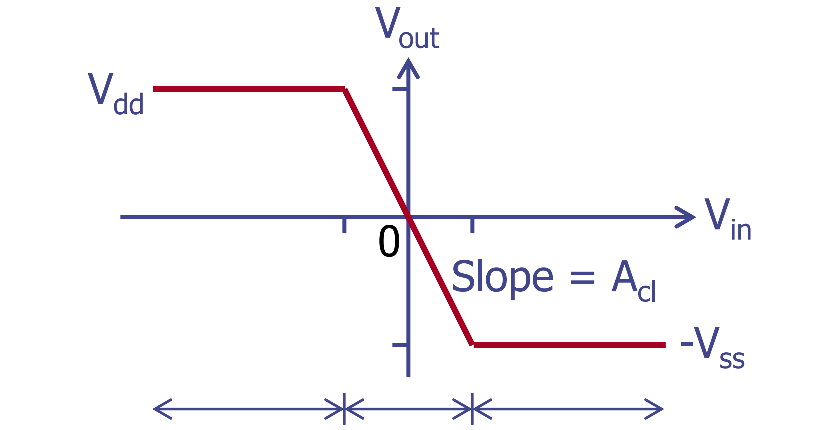
$$I_2 = \frac{V_o}{2k} = \frac{5}{2k} = 2.5mA$$



4.2.2 Inverting Amplifier

Another simple op amp circuit is the inverting amplifier:





Now, $V_- = V_+ = 0\text{V}$

As V_ has the same potential as ground, but itself is not ground, hence, it is called the virtual ground.

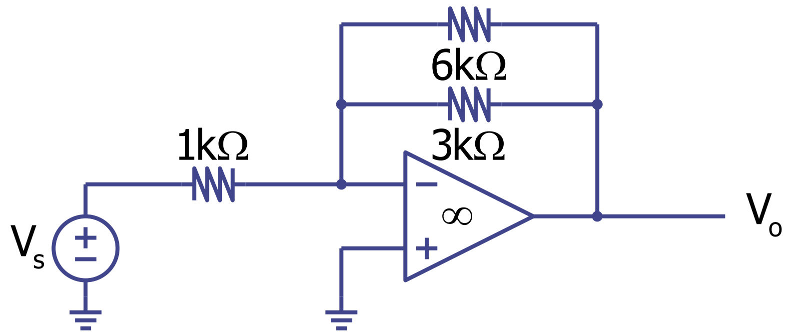
Next, $$I = \frac{V_{in} - V_-}{R_1} = \frac{V_- - V_{out}}{R_2}$$

$$\Rightarrow \quad \frac{V_{out}}{V_{in}} = A_{cl} = -\frac{R_2}{R_1}$$

Note that the polarity of V_{out} is opposite to V_{in}, thus output is inverted.

Example 4-7

Example 4-7: Let the op amp be ideal, (i) find V_o for $V_s = 1$ V; and (ii) find $V_o(t)$ for $V_s(t) = \sin(2\pi 50t)$ V.

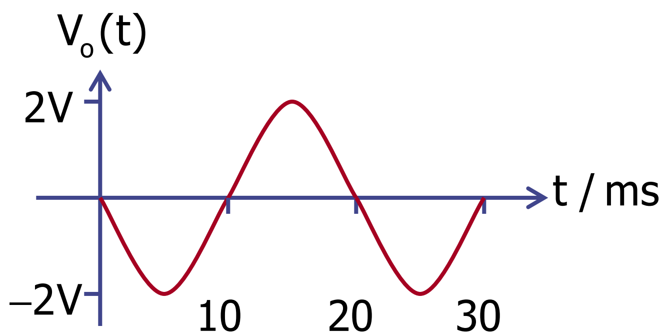


Soln.:

(i) $$\frac{V_o}{V_s} = -\frac{6k\,||\,3k}{1k} = -2$$

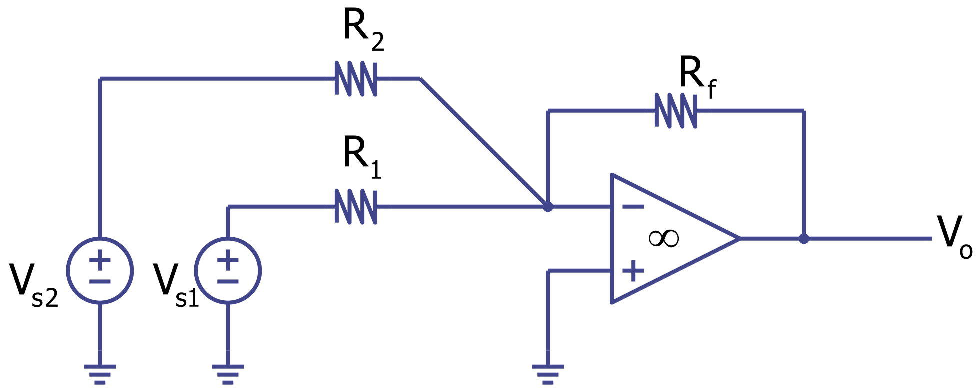
$$\Rightarrow \quad V_o = -2 \times 1 = -2V$$

(ii) $$V_o(t) = -2\sin(2\pi 50t)\ V$$



4.2.3 Summing Amplifier

By adding more inputs to the inverting amplifier, a summing amplifier can be realized.



By noting that $V_{-} = 0$ we have

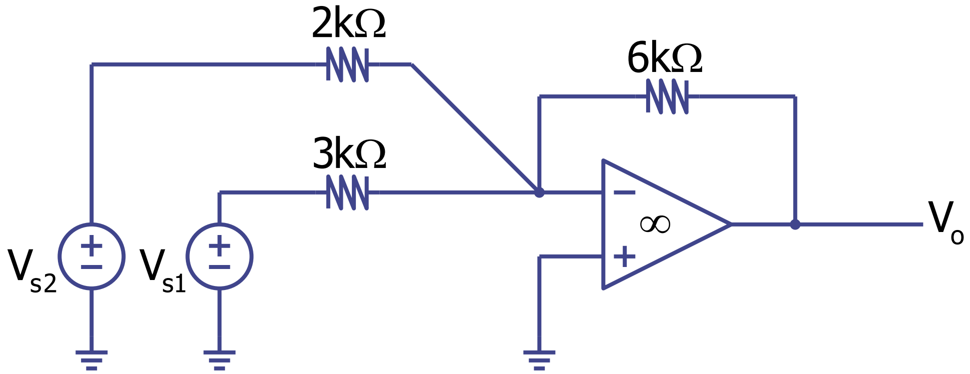
$$\frac{V_{s1}}{R_1} + \frac{V_{s2}}{R_2} = \frac{0 - V_o}{R_f}$$

$$\Rightarrow \quad V_o = -\frac{R_f}{R_1} V_{s1} - \frac{R_f}{R_2} V_{s2}$$

The output voltage V_o is the weighted sum of the inputs V_{s1} and V_{s2}.

Example 4-8

Example 4-8: For the summing amplifier shown below, find V_o for V_{s1} = 1 V and (1) V_{s2} = 2 V; and (2) $V_{s2}(t) = 2\sin\omega t$ V.

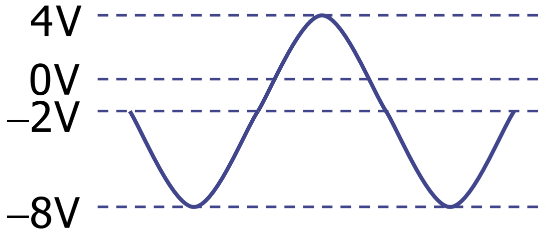


Soln.:

(1) $$V_o = -\frac{6k}{3k}\times 1 - \frac{6k}{2k}\times 2 = -2-6 = -8\text{V}$$

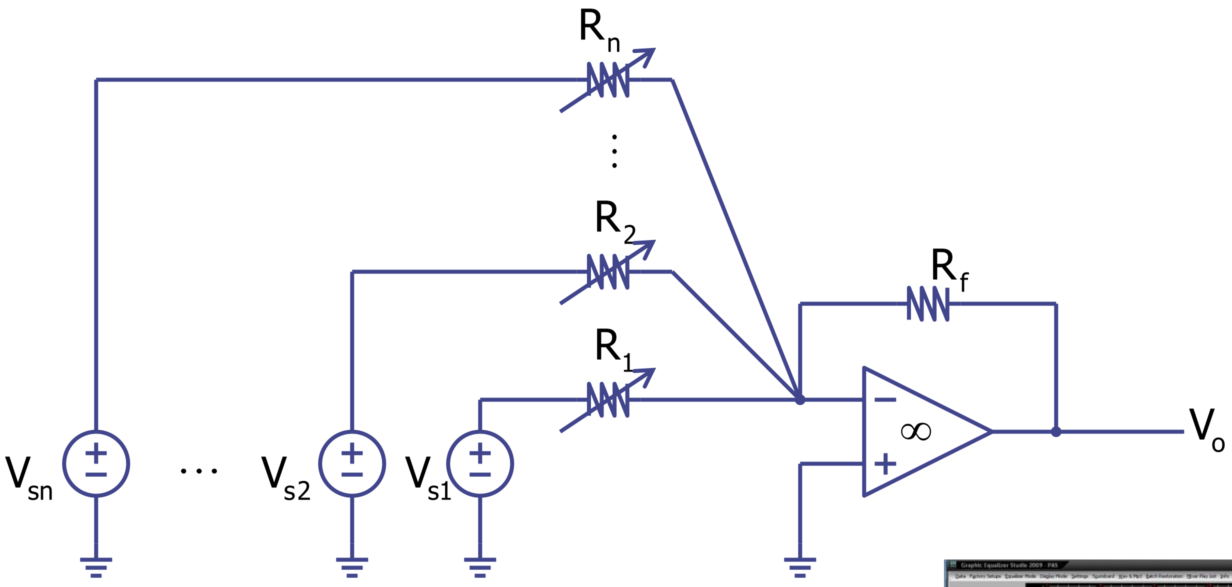
(2) $$V_o(t) = -\frac{6k}{3k}\times 1 - \frac{6k}{2k}\times 2\sin\omega t$$

$$= -2 - 6\sin\omega t$$



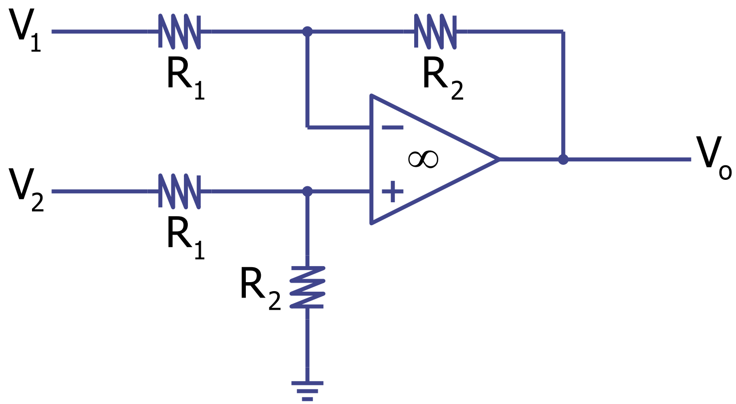
Summing Amplifier as Audio Mixer

By adding more inputs to the summing amplifier, and the gain of each input branch can be adjusted by a variable resistor, an audio mixer can be constructed. Other applications include karaoke, equalizer, synthesizer.



4.2.4 Difference Amplifier

Besides addition, we may arrange to have a difference amplifier.



$$V_+ = V_- = \frac{R_2}{R_1 + R_2} V_2$$

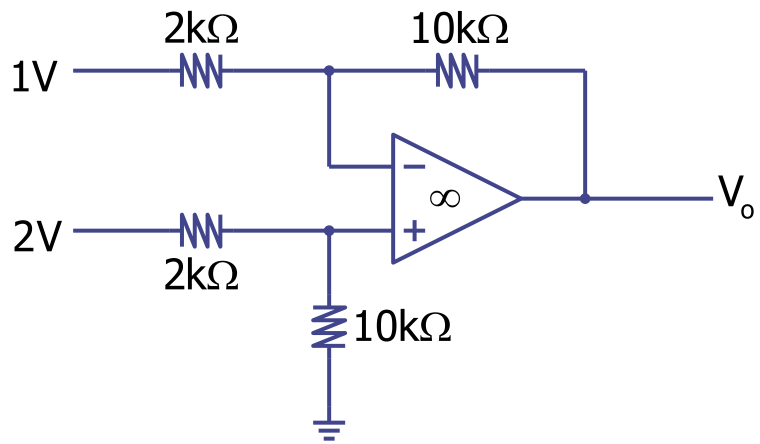
and $$\frac{V_1 - V_-}{R_1} = \frac{V_- - V_o}{R_2}$$

$$\Rightarrow \quad \frac{V_1}{R_1} + \frac{V_o}{R_2} = \frac{R_1 + R_2}{R_1 R_2} V_- = \frac{R_1 + R_2}{R_1 R_2} \frac{R_2}{R_1 + R_2} V_2 = \frac{V_2}{R_1}$$

$$\Rightarrow \quad V_o = \frac{R_2}{R_1}(V_2 - V_1)$$

Example 4-9

Example 4-9: Find V_o of the following difference amplifier.



Soln.:

$$V_o = \frac{10k}{2k}(2-1)$$

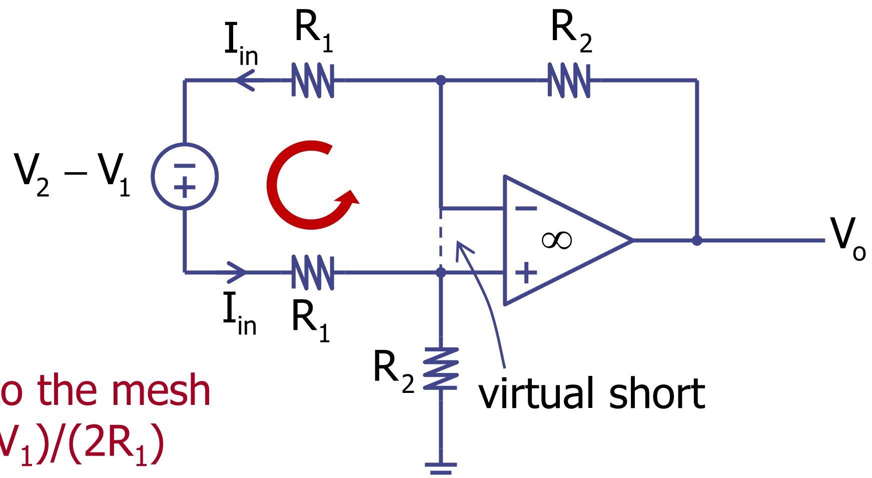
$$= 5V$$

Disadvantages of Difference Amplifier

The difference amplifier has two disadvantages:

(1) To change the gain of the amplifier, two resistors (either all R_1 or all R_2) have to be changed.

(2) An ideal voltage amplifier has infinite input resistance. Now, the input resistance is only $2R_1$, and it draws considerable current from the input $(V_2–V_1)$. Moreover, if a larger gain is achieved by lowering R_1's, a larger current is drawn from $(V_2–V_1)$.

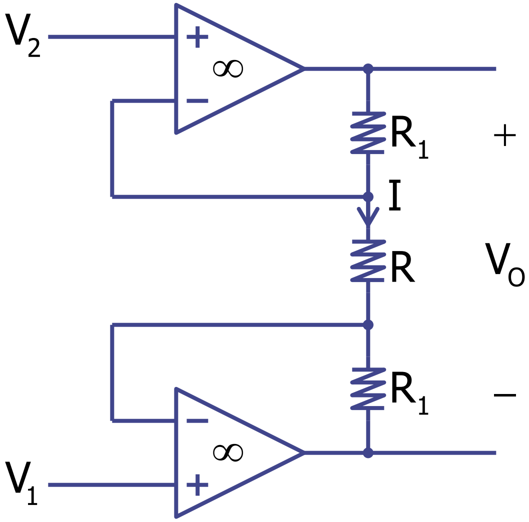


Apply KVL to the mesh
$I_{in}=(V_2 - V_1)/(2R_1)$

4.2.5 Instrumentation Amplifier

A difference amplifier can be realized by the instrumentation amplifier:

$$
\begin{aligned}
V_o &= I \times (R + 2R_1) \\
&= \frac{V_2 - V_1}{R} \times (R + 2R_1) \\
&= \left(1 + \frac{2R_1}{R}\right) \times (V_2 - V_1)
\end{aligned}
$$



Advantages:

(1) The input resistance is very high, and no current is drawn from the sources.

(2) The gain can be adjusted by changing one resistor (R).

Disadvantage:

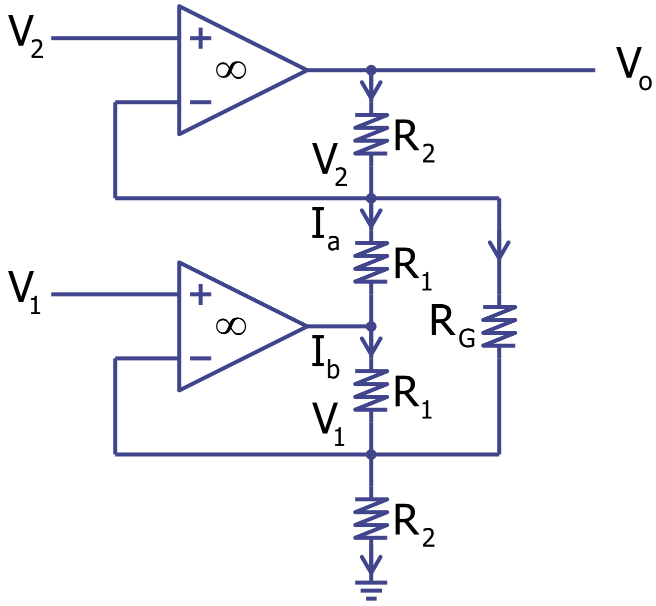
The output is a differential voltage.

IA with Single-Ended Output

The previous instrumentation amplifier can be modified to have singled-ended output by adding two extra resistors:

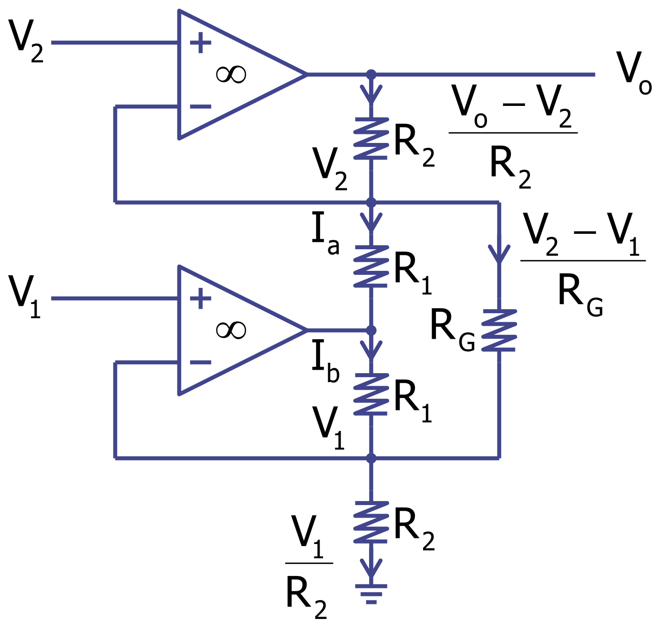
$$V_2 = I_a R_1 + I_b R_1 + V_1$$

$$\Rightarrow \frac{V_2 - V_1}{R_1} = I_a + I_b$$



IA with Single-Ended Output (cont.)

The previous instrumentation amplifier can be modified to have singled-ended output by adding two extra resistors:



$$
\begin{aligned}
V_2 &= I_a R_1 + I_b R_1 + V_1 \\
\Rightarrow \frac{V_2 - V_1}{R_1} &= I_a + I_b \\
&= \frac{V_o - V_2}{R_2} - \frac{V_2 - V_1}{R_G} + \frac{V_1}{R_2} - \frac{V_2 - V_1}{R_G} \\
&= \frac{V_o}{R_2} - \frac{V_2 - V_1}{R_2} - \frac{2(V_2 - V_1)}{R_G} \\
\Rightarrow \frac{V_o}{R_2} &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{2}{R_G} \right)(V_2 - V_1) \\
\Rightarrow V_o &= \left(1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G} \right)(V_2 - V_1)
\end{aligned}
$$

IA with Single-Ended Output (cont.)

Advantages:

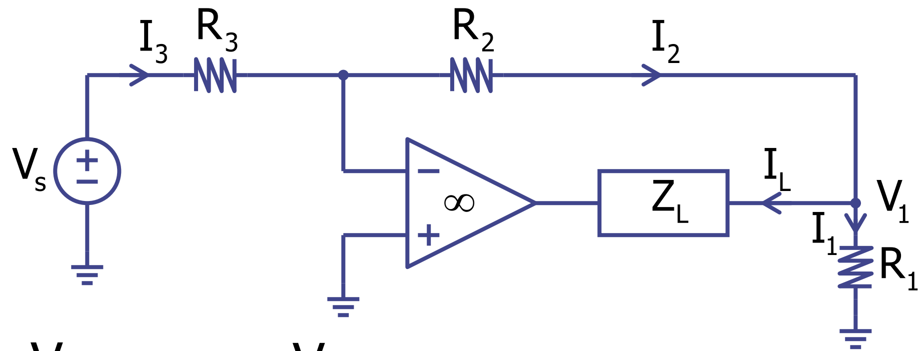
(1) The input resistance is very large (close to ∞), and draws negligible current from the source.

(2) We only need to change R_G to adjust the voltage gain.

(3) The output voltage V_o is now referenced to ground.

4.2.6 Current Source

The following circuit provides a current source that is independent of the load resistance/impedance:



$$I_3 = \frac{V_s}{R_3} = I_2 = -\frac{V_1}{R_2}$$

$$\Rightarrow \quad V_1 = -\frac{R_2}{R_3} V_s$$

$$I_L = I_2 - I_1 = -\frac{V_1}{R_2} - \frac{V_1}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{R_2}{R_3} V_s$$

$$= \left(1 + \frac{R_2}{R_1}\right)\frac{V_s}{R_3}$$

Op Amp Based Current Source (cont.)

From the analysis, the current source I_L is independent of the load Z_L. It only depends on the voltage source V_s, and accurate resistors R_1, R_2 and R_3 (but Z_L is not connected to ground).

Example 4-10: Find I_L given V_s=2 V, R_1=10 kΩ, R_2=1 MΩ and R_3=100 kΩ.

Soln.:

$$I_L = \left(1 + \frac{1\text{M}}{10\text{k}}\right)\frac{2}{100\text{k}}$$

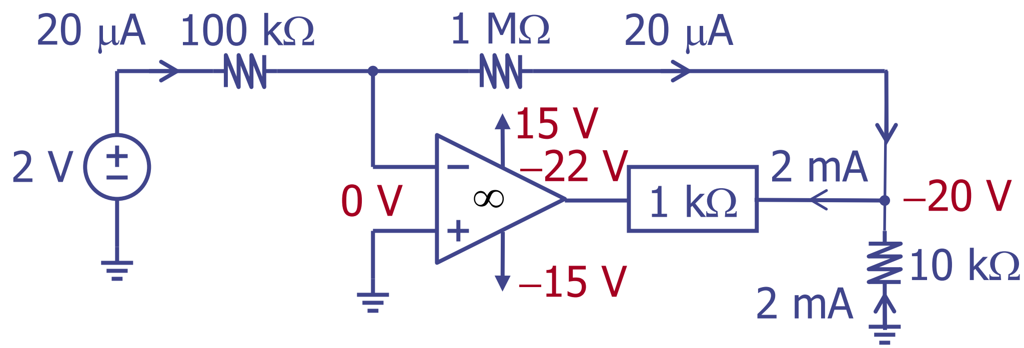
$$= 2.02\text{mA}$$

Note that $I_3 = V_s/R_3 = 2/100$ k = 20 μA and is negligibly small.

So everything seems good? You get 2 mA no matter what Z_L is?

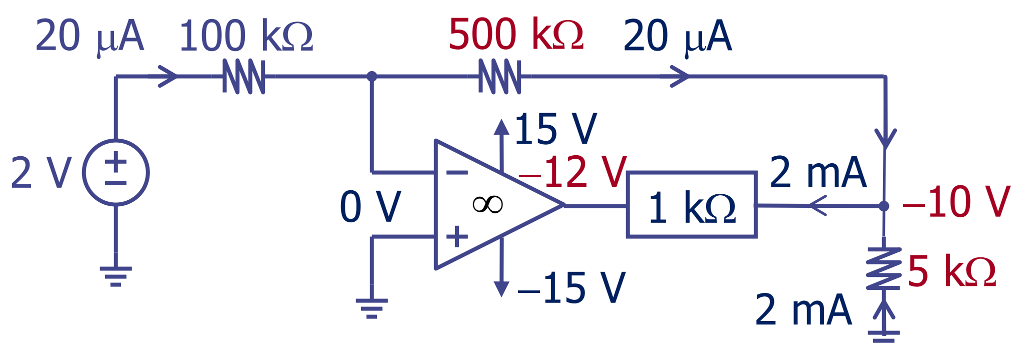
Op Amp Based Current Source – Check Design!

Let's plug in all the actual voltages and currents. Voltages in part of the circuit are outside the ±15 V supply range even for Z_L=0. Moreover, the op am should have saturated and so $V_+ = V_-$ does not hold.



Design doesn't work

Below is an improved design but Z_L is still limited to 2.5 kΩ max.

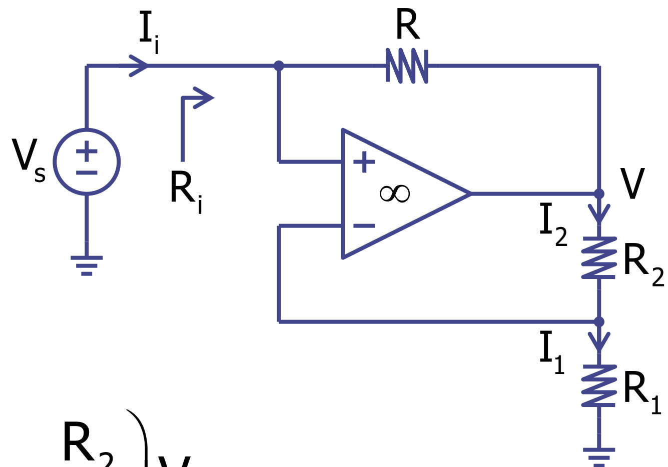


Design OK

Performing design iterations, optimizations and tradeoffs are routine job for a circuit designer.

4.2.7 Negative Impedance Converter

A negative impedance converter realizes the negative value of a specify resistance R or impedance Z.



$I_1 = I_2$

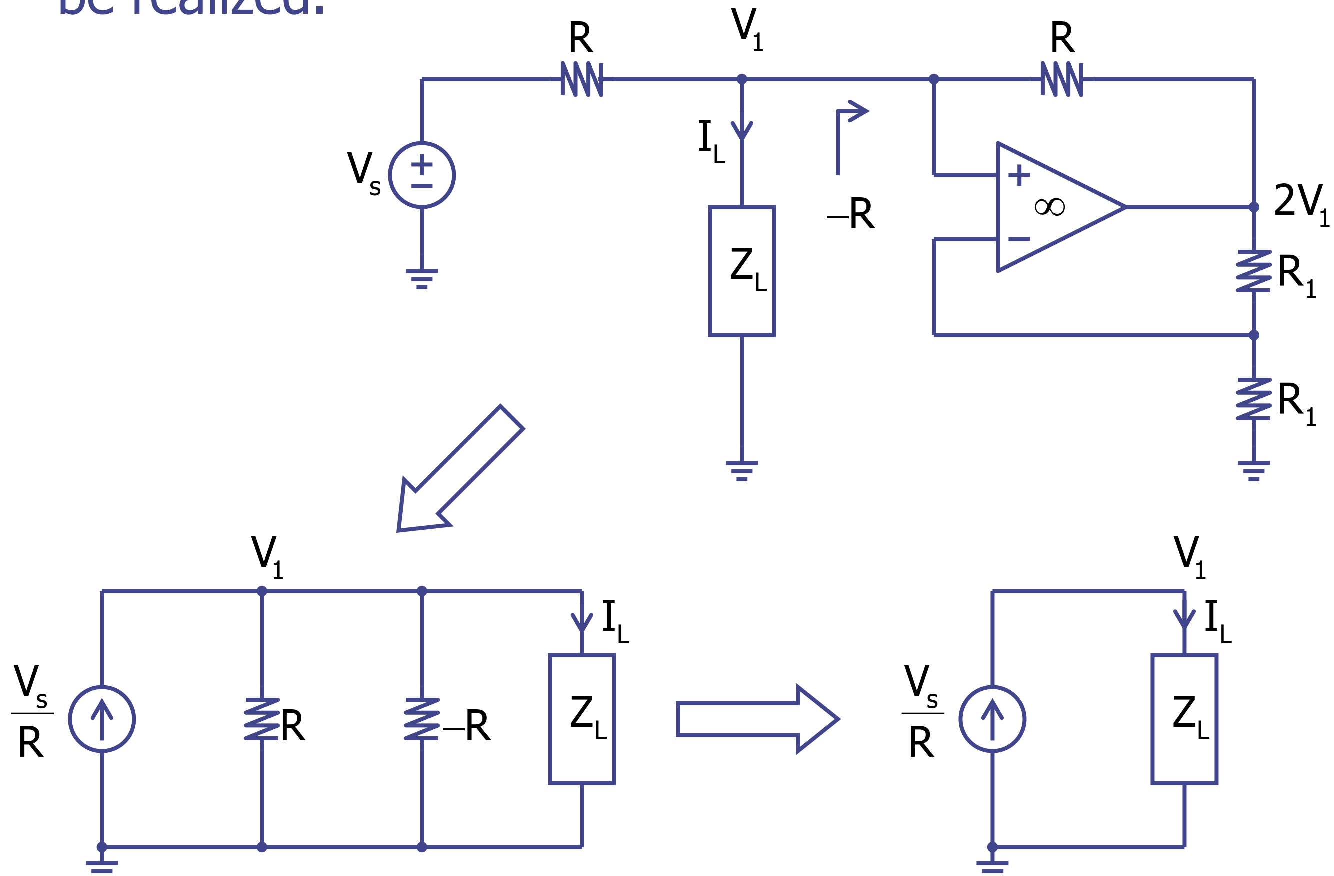
$$V = \left(1 + \frac{R_2}{R_1}\right) V_s$$

$$I_i = \frac{V_s - V}{R} = -\frac{R_2}{R_1}\frac{V_s}{R}$$

$$R_i = \frac{V_s}{I_i} = -\frac{R_1}{R_2} R$$

4.2.8 Voltage-to-Current Converter

Based on the negative impedance converter, a voltage-to-current converter (V-to-I converter) that is independent of the load can be realized.



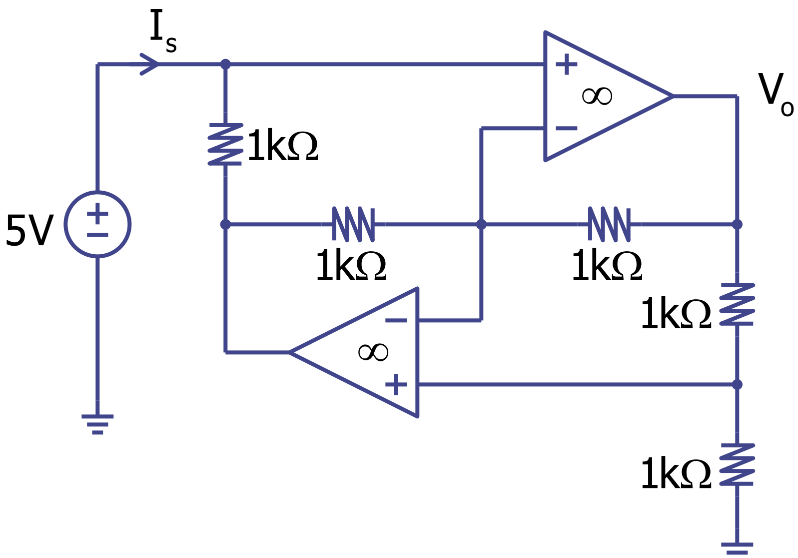
$$I_L = \frac{V_s}{R}$$

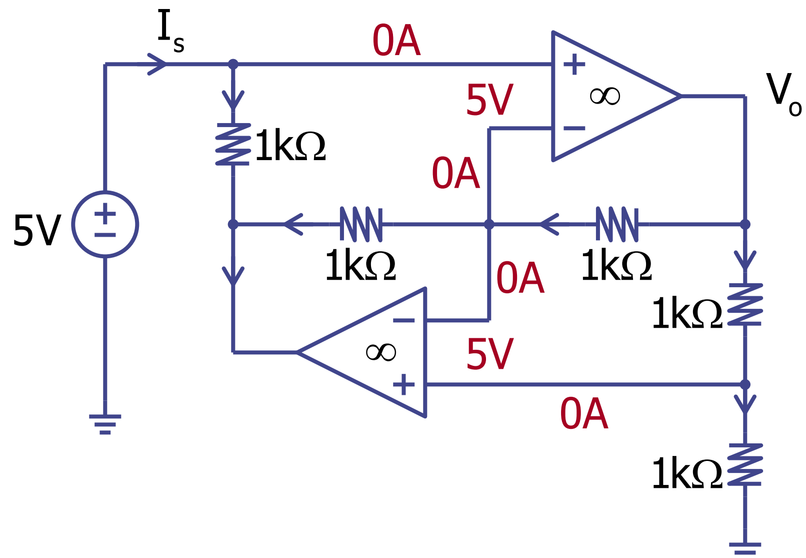
$$V_1 = \frac{Z_L}{R} V_s$$

R||(–R)=∞

Example 4-11a

Example 4-11a: Find V_o and I_s of the following circuit.

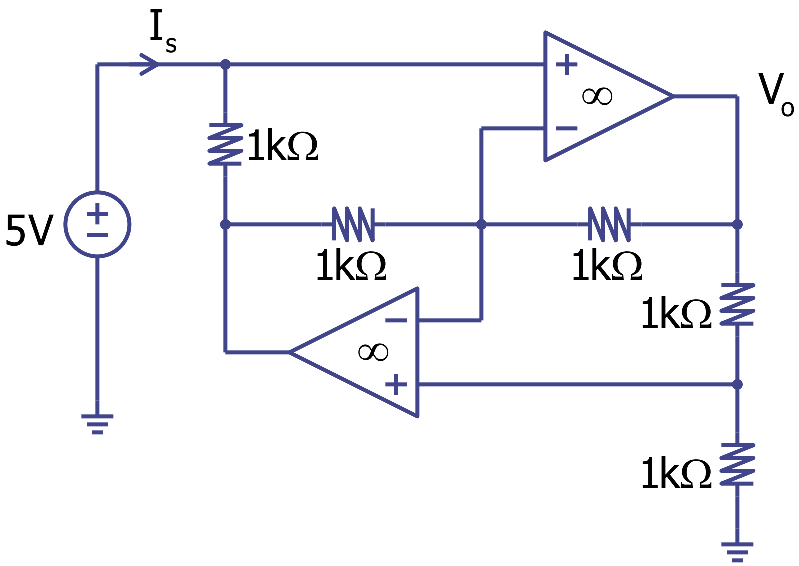


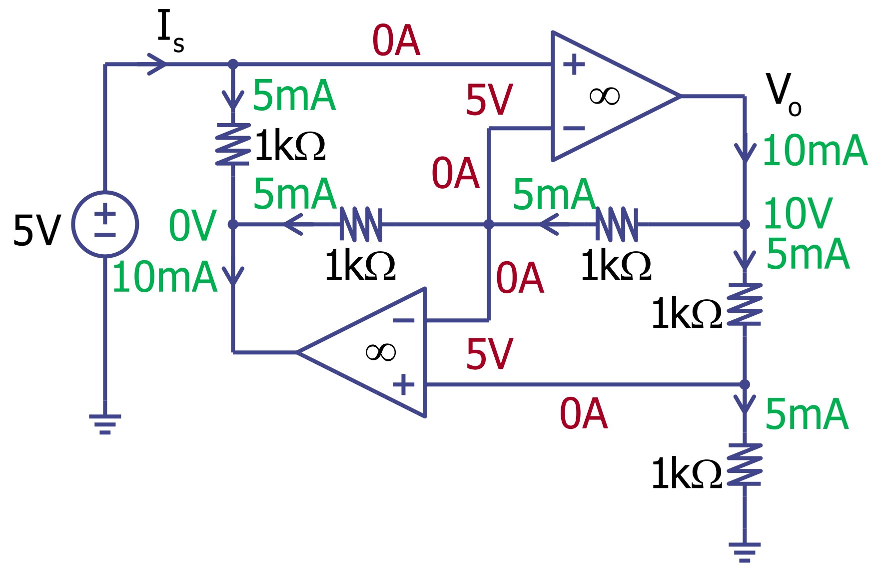


Key observations for this circuit analysis:
- All the input voltages to the op amps are at 5 V.
- All the input currents are zero.

Example 4-11a (cont.)

Example 4-11a: Find V_o and I_s of the following circuit.

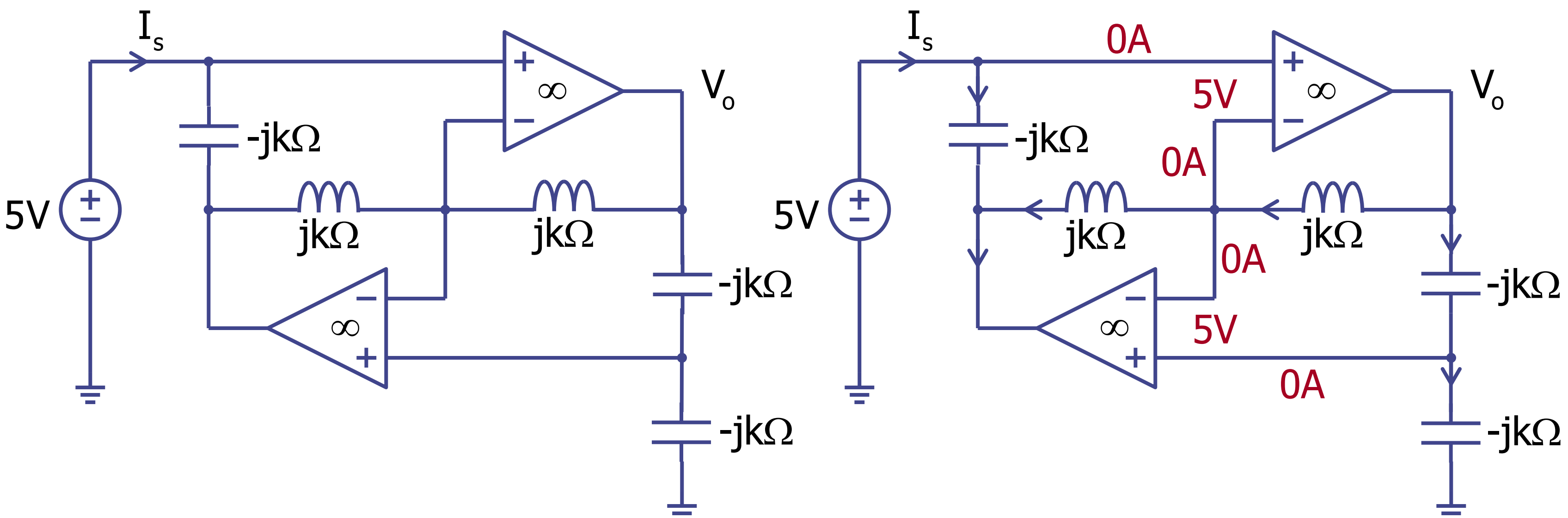




Key observations for this circuit analysis:
- All the input voltages to the op amps are at 5 V.
- All the input currents are zero.
- As a final check, all the voltages seem reasonable.

Example 4-11b (AC Analysis)

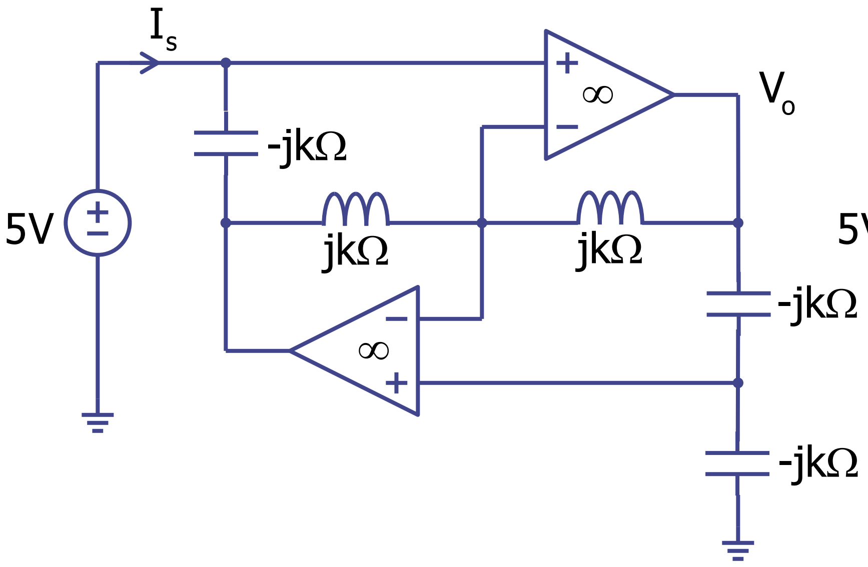
Example 4-11b: Find V_o and I_s of the following circuit.
This is the same circuit as before but with resistors replaced by capacitors and inductors. The excitation in this case is $5\angle 0^o$ V.
Our phasor analysis methods are all applicable here.

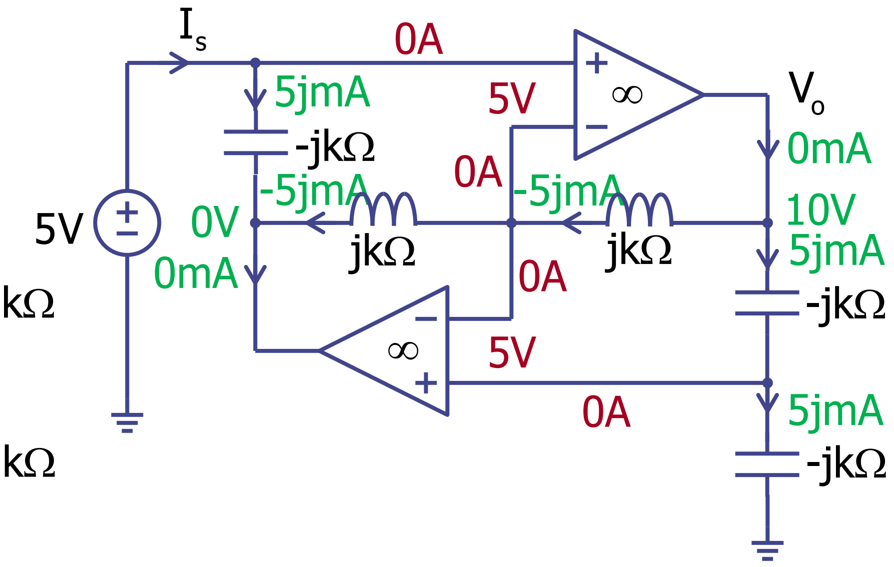


Key observations for this circuit analysis:
- All the input voltages to the op amps are at 5 V.
- All the input currents are zero.

Example 4-11b (cont.)

Example 4-11b: Find V_o and I_s of the following circuit.



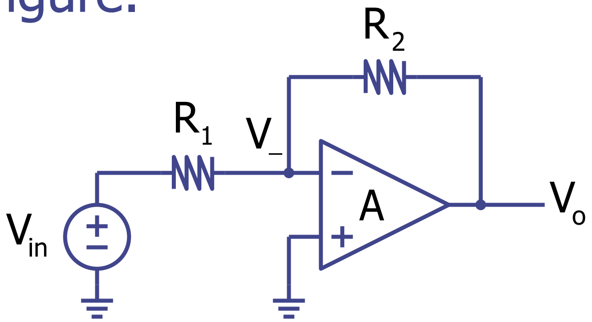


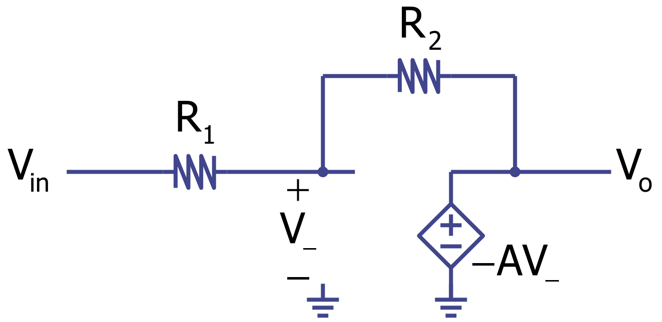
Key observations for this circuit analysis:

- All the input voltages to the op amps are at 5 V.
- All the input currents are zero.
- As a final check, all the voltages seem reasonable.

4.2.9 Effect of Finite Op Amp Gain

For a practical op amp, the open-loop gain A is not infinite. It may be 10^4 V/V (80 dB), 1000 V/V (60 dB), etc. The circuit model of the inverting amplifier with a finite gain A is shown in the right figure.





Effect of Finite Op Amp Gain (cont.)

For $A \neq \infty$, $V_- \neq V_+$, and $V_o = -AV_-$.
Now,

$$\frac{V_{in} - V_-}{R_1} = \frac{V_- - V_o}{R_2}$$

$$\Rightarrow \quad \frac{V_{in}}{R_1} = \frac{-V_o}{R_2} + \frac{V_-}{R_1} + \frac{V_-}{R_2} = \frac{-V_o}{R_2}\left[1 + \frac{1}{A}\left(1 + \frac{R_2}{R_1}\right)\right]$$

$$\Rightarrow \quad \frac{V_o}{V_{in}} = A_{cl} = -\frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{1}{A}\left(1 + \frac{R_2}{R_1}\right)}$$

For $A >> 1$, then

$$\frac{V_o}{V_{in}} \approx -\frac{R_2}{R_1}$$

Example 4-12

Example 4-12: Let R_1 = 1 kΩ and R_2 = 10 kΩ. Compute the closed-loop gain A_{cl} of the inverting amplifier for A = 100, 1000, 10^4 and 10^5.

Soln.:

$$A_{cl}\Big|_{A=10^2} = -10\cdot\frac{1}{1+\frac{1}{100}(11)} = \frac{-10}{1.11} = -9.01 \text{ V/V}$$

$$A_{cl}\Big|_{A=10^3} = -10\cdot\frac{1}{1+\frac{1}{1000}(11)} = \frac{-10}{1.011} = -9.89 \text{ V/V}$$

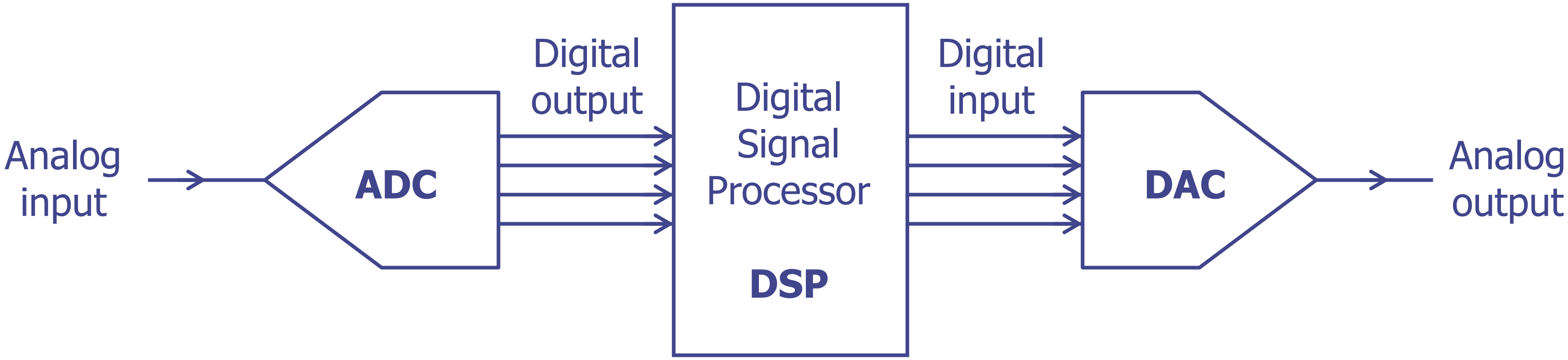
$$A_{cl}\Big|_{A=10^4} = -10\cdot\frac{1}{1+\frac{1}{10000}(11)} = \frac{-10}{1.0011} = -9.99 \text{ V/V}$$

$$A_{cl}\Big|_{A=10^5} = -10\cdot\frac{1}{1+\frac{1}{100000}(11)} = \frac{-10}{1.00011} = -10.0 \text{ V/V}$$

Chapter 4: Operational Amplifers and Circuits

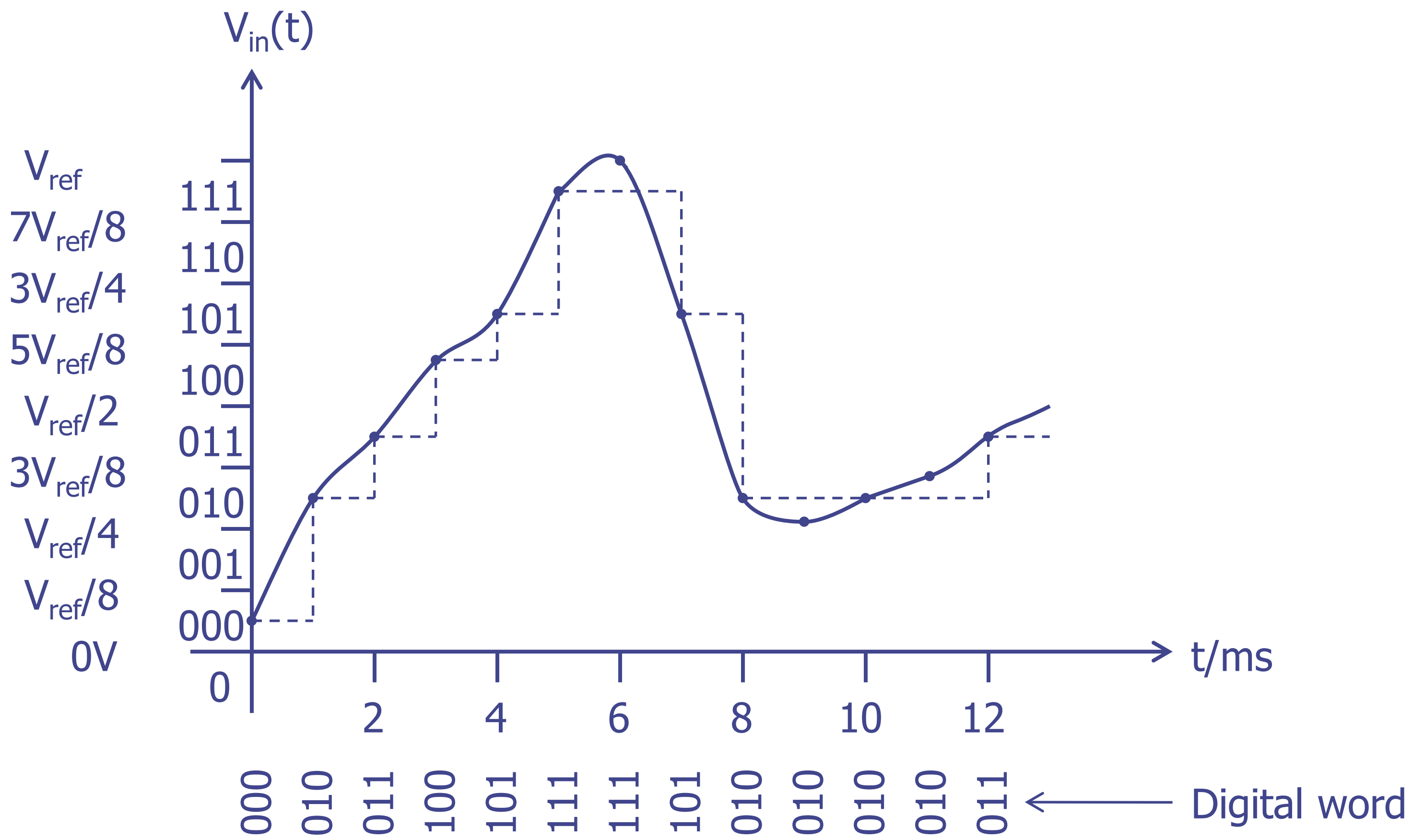
4.3 Signal Processing Applications

Signals such as voice and temperature vary continuously with time, and are known as analog signals. With the exploding development of digital computers, it is more efficient to convert an analog signal into its digital equivalent for processing (analog-to-digital conversion, or A/D conversion), and then convert it back to analog signal as output (digital-to-analog conversion, D/A conversion). The circuit that performs A/D conversion is an analog-to-digital converter (ADC), and that performs D/A conversion is a digital-to-analog converter (DAC).



4.3.1 Analog-to-Digital Converter

Digitizing an analog signal to a digital signal with 3-bit resolution.

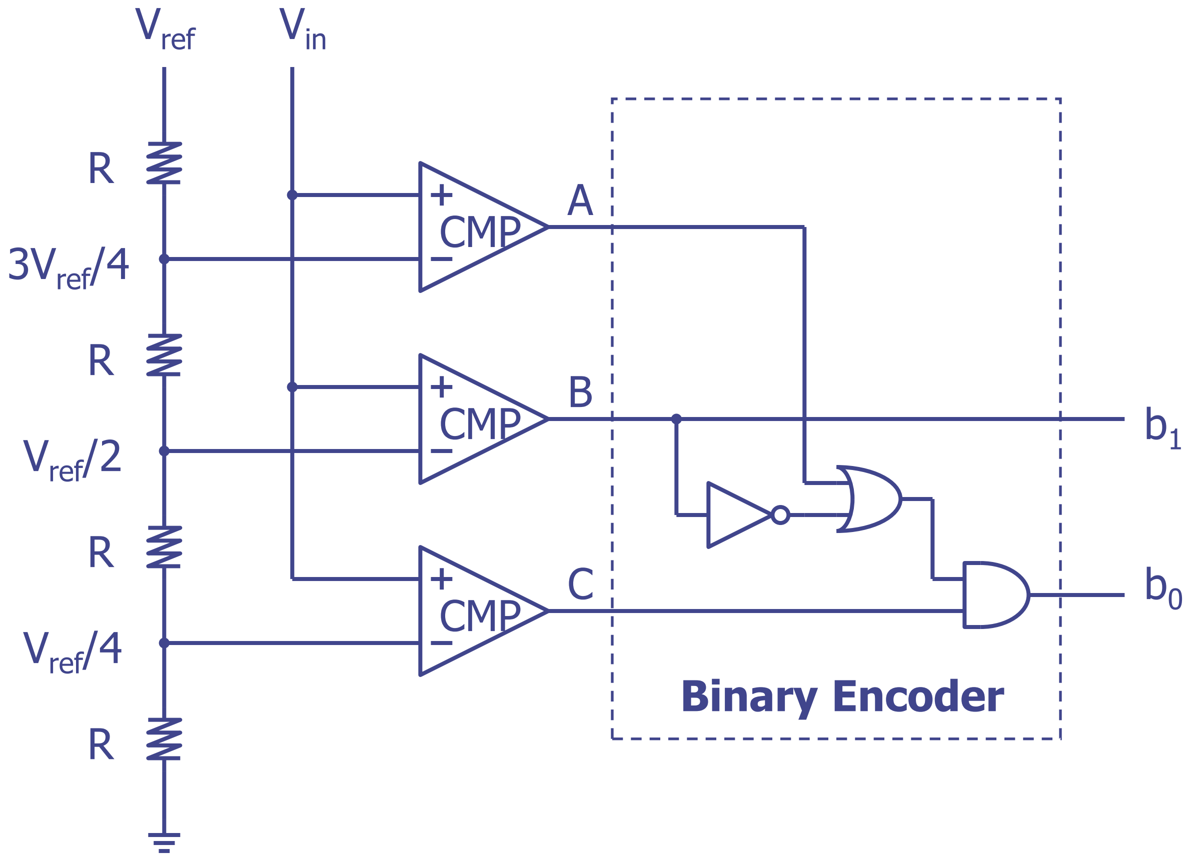


Note that the highest-frequency component representable is fs/2, where fs is the sampling frequency.

https://en.wikipedia.org/wiki/Nyquist_frequency

2-Bit Flash ADC

Using comparators (op amps in open loop) and logic gates, a 2-bit flash analog-to-digital converter can be constructed.



2-Bit Flash ADC (cont.)

A flash ADC can make one conversion per clock cycle (instead of one conversion per several clock cycles). A 2-bit flash ADC needs 3 (= $2^2 - 1$) comparators, and an n-bit flash ADC needs $2^n - 1$ comparators.

Analog Signal	A	B	C	b_1	b_0
$V_i < \frac{1}{4}V_{ref}$	0	0	0	0	0
$\frac{1}{4}V_{ref} < V_i < \frac{1}{2}V_{ref}$	0	0	1	0	1
$\frac{1}{2}V_{ref} < V_i < \frac{3}{4}V_{ref}$	0	1	1	1	0
$\frac{3}{4}V_{ref} < V_i$	1	1	1	1	1

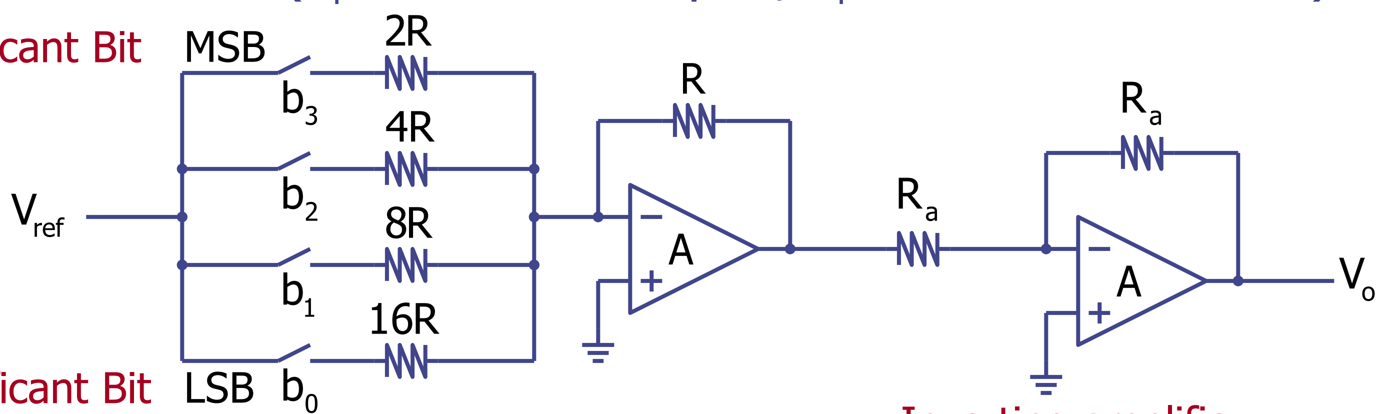
By considering "don't care" conditions, it can be shown that

$$b_1 = B$$
$$b_0 = (A+\overline{B})C$$

4.3.2 Digital-to-Analog Converter

A 4-bit DAC can be constructed using the summing amplifier as shown below. (b_i=0 $\Rightarrow$ switch open, b_i=1 $\Rightarrow$ switch closed)

Most Significant Bit



Least Significant Bit

Inverting amplifier

$$V_o = \left(\frac{b_3}{2R}+\frac{b_2}{4R}+\frac{b_1}{8R}+\frac{b_0}{16R}\right)V_{ref}\times R = \left(\frac{b_3}{2}+\frac{b_2}{4}+\frac{b_1}{8}+\frac{b_0}{16}\right)V_{ref}$$

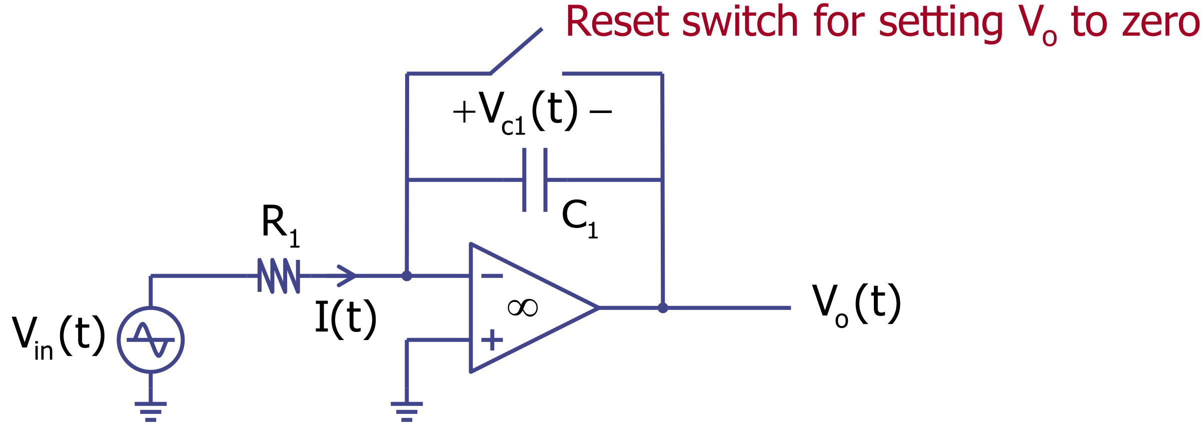
Let V_{ref}=2 V, For $b_3b_2b_1b_0$ = 1010,

$$V_o = \left(\frac{1}{2}+\frac{0}{4}+\frac{1}{8}+\frac{0}{16}\right)\times 2 \;=1.25\text{V}$$

CD player

4.3.3 Integrator and Differentiator

An op amp can be connected to become an integrator:



$$\frac{V_{in}(t)}{R_1} = I(t) = C_1 \frac{dV_{c1}(t)}{dt} = -C_1 \frac{dV_o(t)}{dt}$$

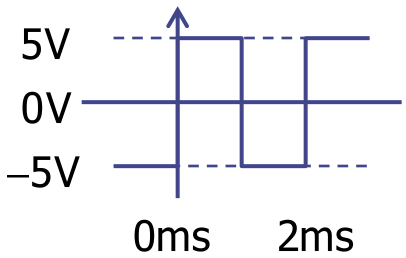
$$\Rightarrow \quad V_o(t) = -\frac{1}{C_1 R_1} \int_0^t V_{in}(\lambda) d\lambda + V_o(0)$$

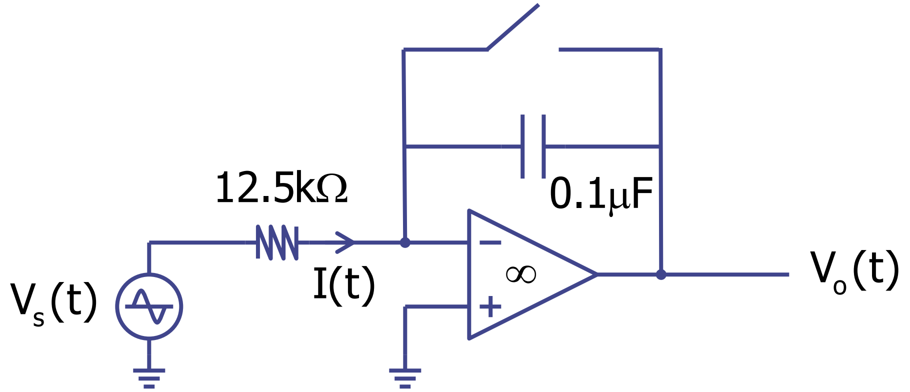
Resettable

Apart from the integration constant, the output waveform is the integral of the input waveform.

Example 4-13

Example 4-13: Find $V_o(t)$ given $V_o(0) = 2$ V.



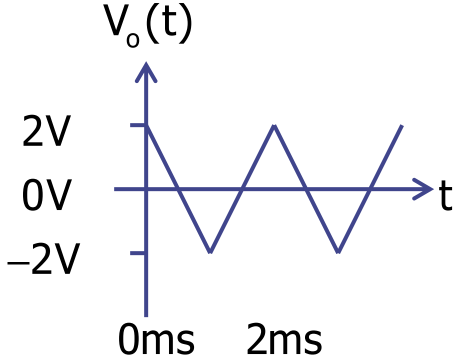


Soln.:

$$V_o(t) = -\frac{1}{0.1\mu \times 12.5k}\int_0^t V_{in}(\lambda)d\lambda + 2$$

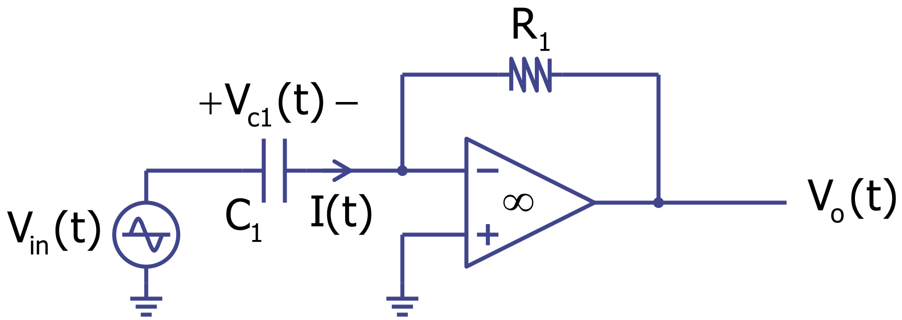
$$V_o(1ms) = -\frac{1}{1.25m}\int_0^{1m} 5d\lambda + 2$$

$$= -4 + 2 \quad = -2V$$



Differentiator

An op amp can be connected to become a differentiator:

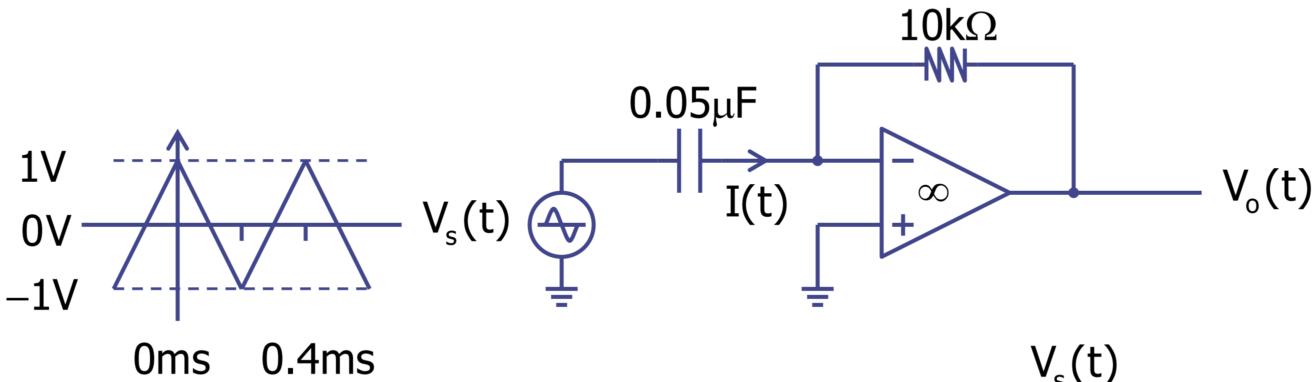


$$\frac{-V_o(t)}{R_1} = I(t) = C_1 \frac{dV_{c1}(t)}{dt} = C_1 \frac{dV_{in}(t)}{dt}$$

$$\Rightarrow \quad V_o(t) = -C_1 R_1 \frac{dV_{in}(t)}{dt}$$

Example 4-14

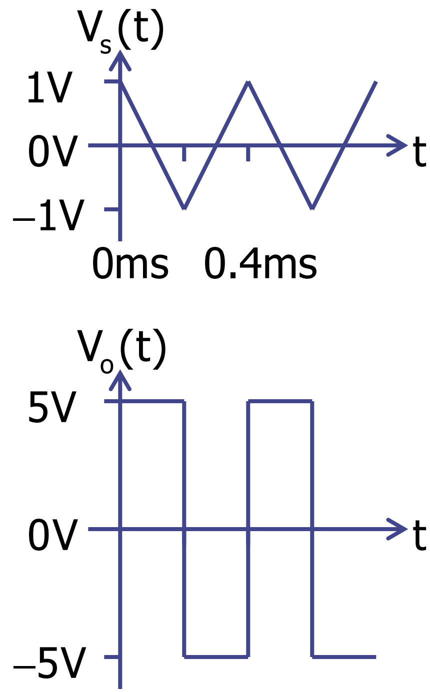
Example 4-14: Find and plot $V_o(t)$.



Soln.:

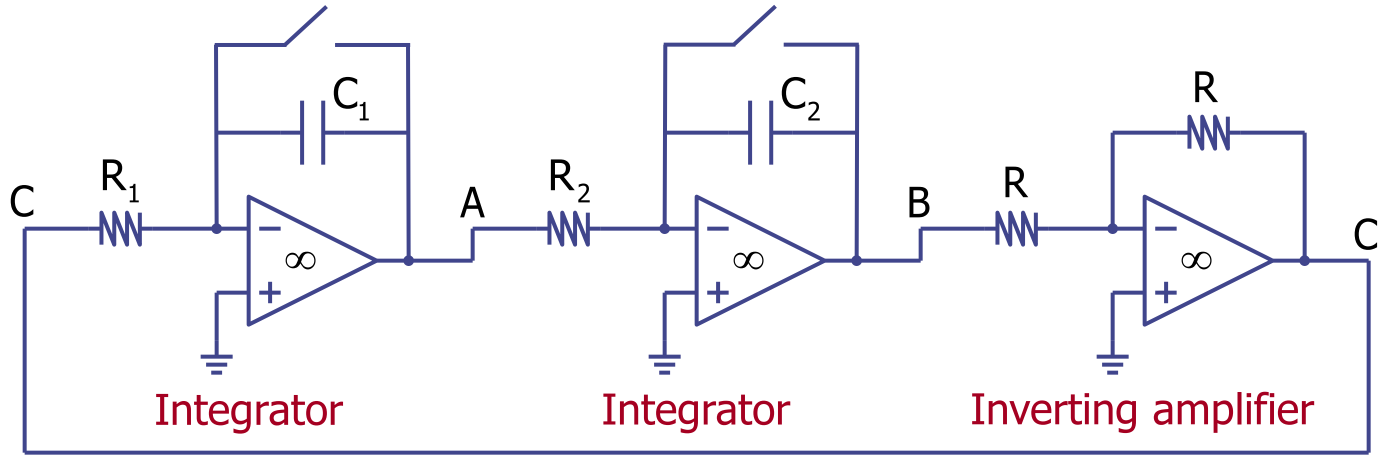
From t=0 to t=0.2 ms, the input voltage has a constant slope of –2 V/0.2 ms = 10,000V/s, and

$$V_o(t) = -0.05\mu \times 10k \frac{-1-1}{0.2m} = 5V$$



4.3.4 Oscillator

Using two integrators in cascade, an oscillator with natural (undamped) frequency defined by the R's and C's can be obtained.



$$V_C(t) = -R_1C_1\frac{dV_A(t)}{dt}$$

$$V_A(t) = -R_2C_2\frac{dV_B(t)}{dt}$$

$$V_C(t) = -V_B(t)$$

Oscillator (cont.)

Clearly,

$$V_B(t) = -V_C(t) = R_1C_1\frac{dV_A(t)}{dt}$$

$$= -R_2C_2R_1C_1\frac{d^2V_B(t)}{dt^2}$$

Define

$$\omega_o^{\ 2} = \frac{1}{C_1R_1C_2R_2}$$

$$\Rightarrow \quad \frac{d^2V_B(t)}{dt^2} + \omega_o^{\ 2}V_B(t) = 0$$

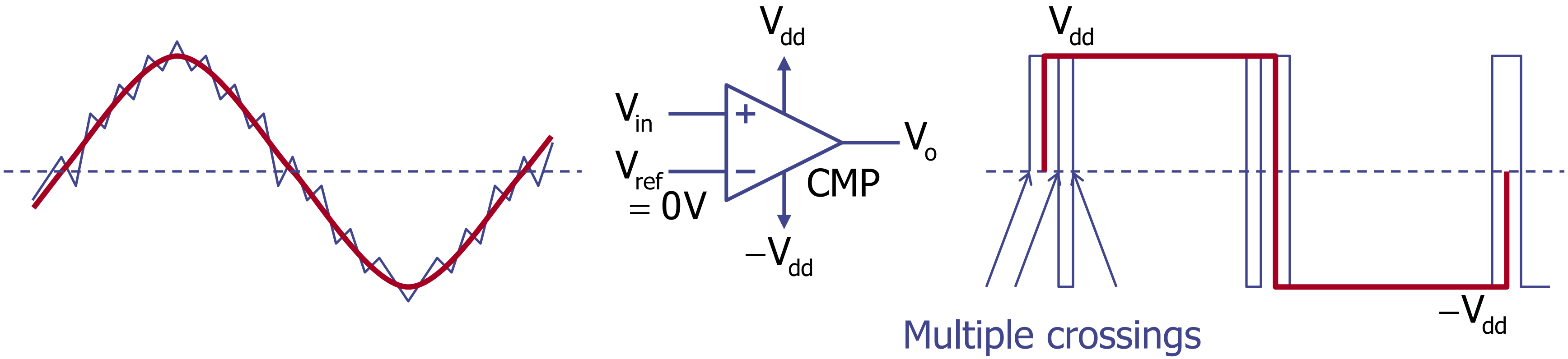
The solution gives an oscillator:

$$V_B(t) = \alpha\cos(\omega_o t + \beta)$$

Note that there is no need for an input, and the op amp circuit can just oscillate by itself, e.g., triggered by noise, due to an overall positive feedback round the loop.

Comparator with Noisy Input

Consider a comparator with a noisy input:

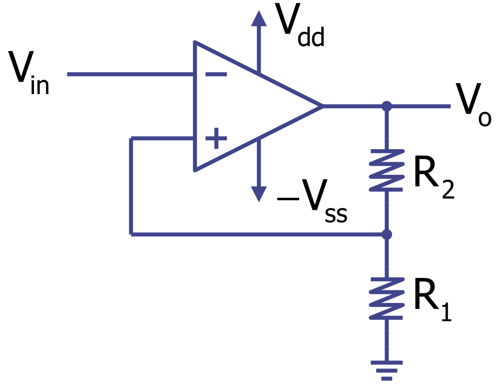


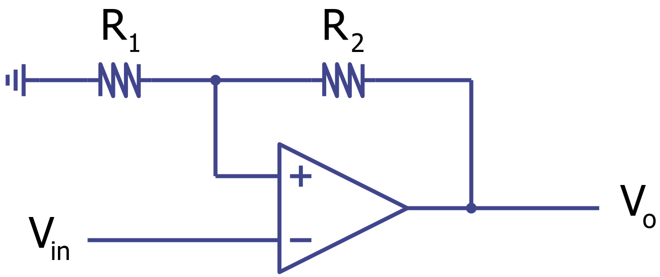
If the comparator is very fast, it will response to noise near $V_{ref} = 0$ immediately and results in multiple crossings before settling down to the required value.

4.3.5 Bistable Circuits

A bistable circuit has two stable states. It can be generated by using positive feedback around an op amp.

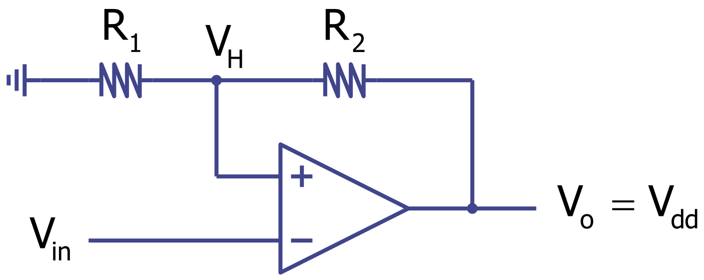
Consider swapping the V_+ and V_- of the non-inverting amplifier such that negative feedback is turned into positive feedback. As such, the output voltage will saturate at either the positive power rail (V_{dd}) or the negative power rail ($-V_{ss}$). An alternative way to draw the circuit is shown to the right.

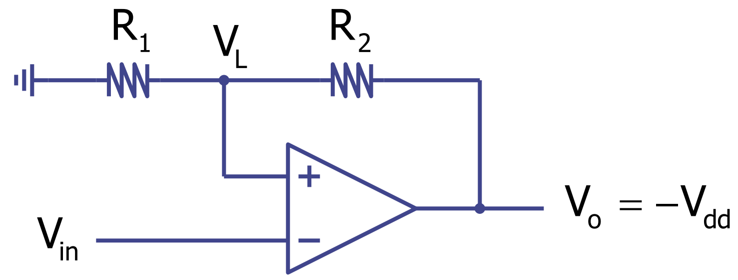




Schmitt Trigger

The circuit is called a Schmitt trigger. For simplicity, let $V_{ss} = V_{dd}$.





Consider the case when $V_{in} = -V_{dd}$, giving $V_o = V_{dd}$ and

$$V_+ = +\frac{R_1}{R_1 + R_2} V_{dd} = V_H$$

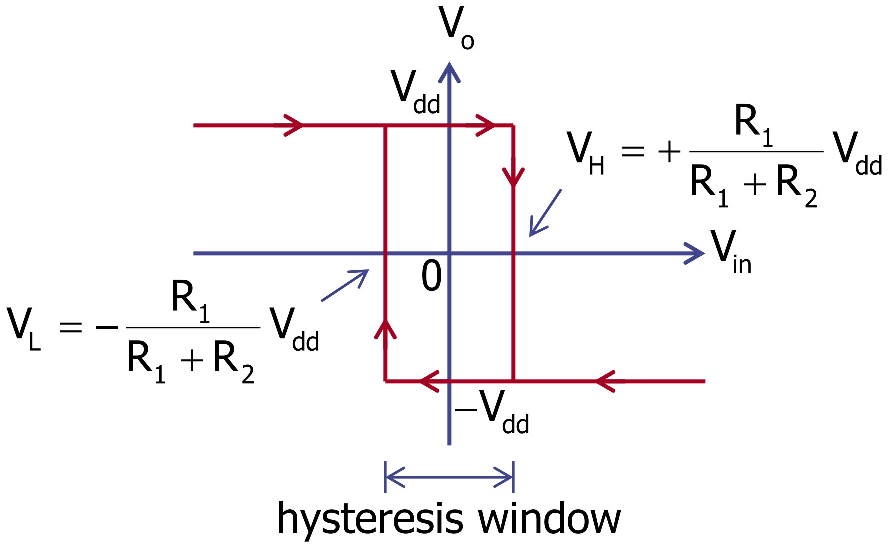
As long as $V_{in} < V_H$ (high threshold), V_o remains at $+V_{dd}$. However, when $V_{in} > V_H$, V_o trips to $-V_{dd}$, and

$$V_+ = -\frac{R_1}{R_1 + R_2} V_{dd} = V_L$$

Similarly, for $V_{in} > V_L$ (low threshold), V_o remains at $-V_{dd}$.

Schmitt Trigger: Hysteresis

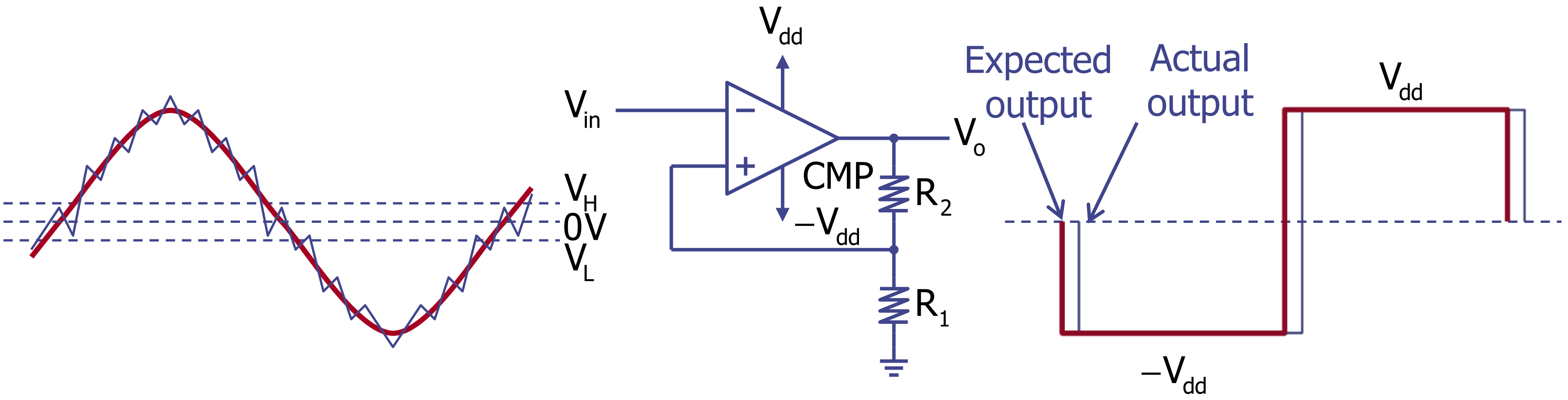
The DC transfer characteristic can be constructed accordingly. It exhibits hysteresis that can be adjusted by R_1:



The quantity $\Delta V_{hys} = V_H - V_L$ is defined as the hysteresis window.

Schmitt Trigger Rejects Noise

Let $V_{ref} = 0$. By designing the hysteresis window appropriately, multiple crossings are avoided, although the output waveform is shifted as $V_H \neq V_L \neq 0$.



Square Waveform Generator

By modifying the Schmitt trigger appropriately, that is,

(1) making the hysteresis window ΔV_{hys} large; and

(2) using the output of the comparator to drive an RC circuit; then we can construct a square waveform generator.

If RC time constant is much larger than the period, a triangular waveform can be generated.