ELEC1100: Introduction to Electro-Robot Design

Lecture 9: Kirchhoff's Laws

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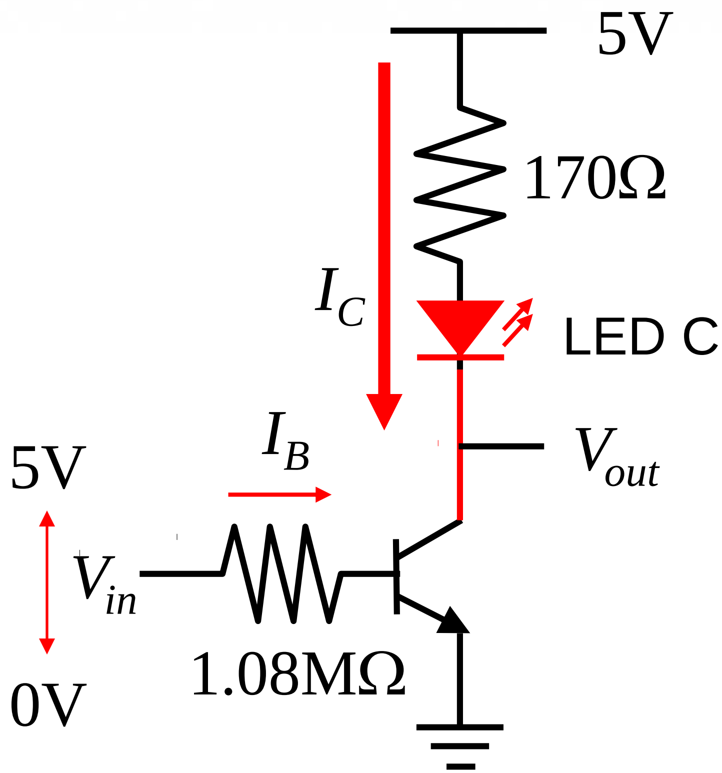
FROM LAST LECTURE

- Circuit analysis: finding I_B and I_C in transistor circuit

$$I_B \approx \frac{V_{in} - 0.7V}{1.08M\Omega}$$

with V_{in}=5V

$$I_B \approx \frac{5V - 0.7V}{1.08M\Omega} = 4\mu A$$



$$I_{Cmax} \approx \frac{5V - 0.7V}{170\Omega} = 25mA$$

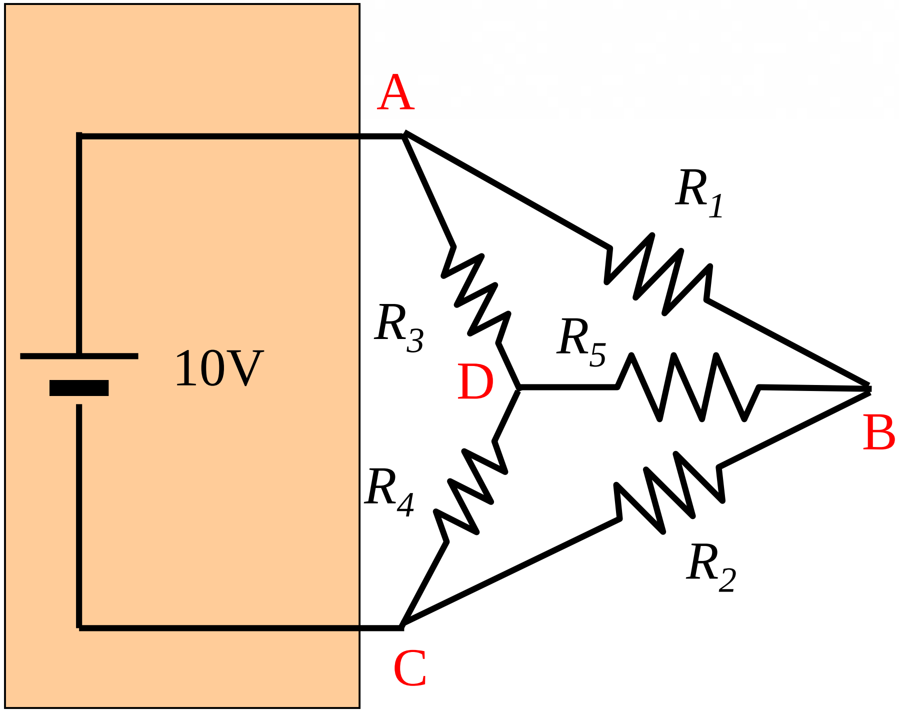
$$I_C = \beta I_B = 0.4mA$$

COMPLEX RESISTOR NETWORK

- A general circuit can be very complicated
- Learn systematic way to analyze circuit like this

Kirchhoff's Current Law

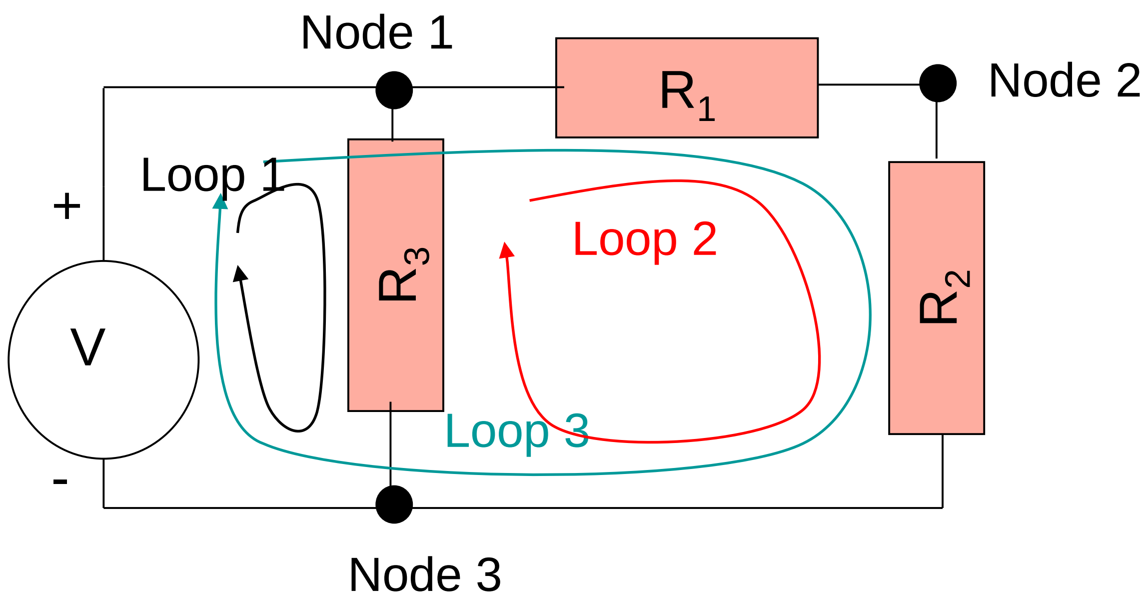
Kirchhoff's Voltage Law



consists of 4 nodes and 6 branches

NODES AND LOOPS

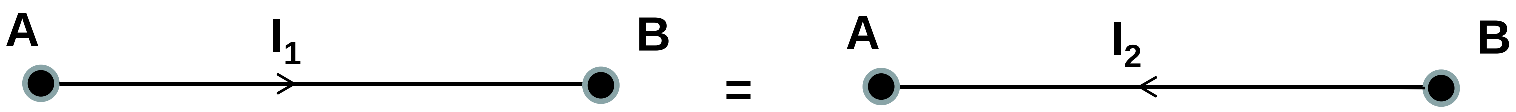
- **Node**: all points connected by a wire, has the same voltage anywhere on the wire
- **Circuit branch**: circuit element between 2 nodes



- **Loop**: any circuit branch that ends at the same starting node

CURRENT AND VOLTAGE DIRECTIONS

- If current flows in one direction is defined as +ve, flow in the other direction is –ve.



$I_1 = I_{AB}$ = current flow from A to B

$I_2 = I_{BA}$ = current flow from B to A

$I_1 = -I_2$ $\quad I_{AB} = -I_{BA}$

- Voltage drop also has direction

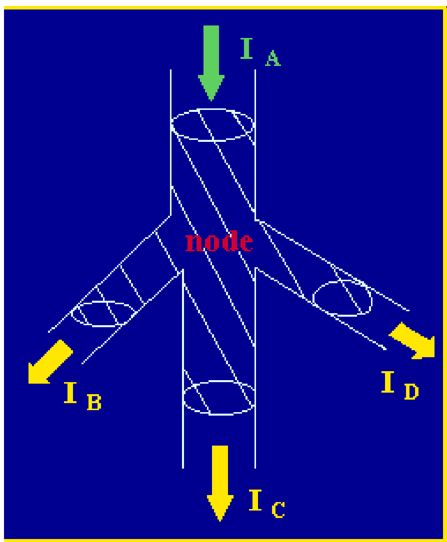
V_{AB} = voltage drop from Node A to Node B

V_{BA} = voltage drop from Node B to Node A

$V_{AB} = -V_{BA}$

KIRCHHOFF'S CURRENT LAW (KCL)

- Electrons can neither be created or destroyed; if it leaves someplace, it has to go to somewhere else
- The algebraic sum of all branch **currents** entering and leaving a node is zero at all instants of time



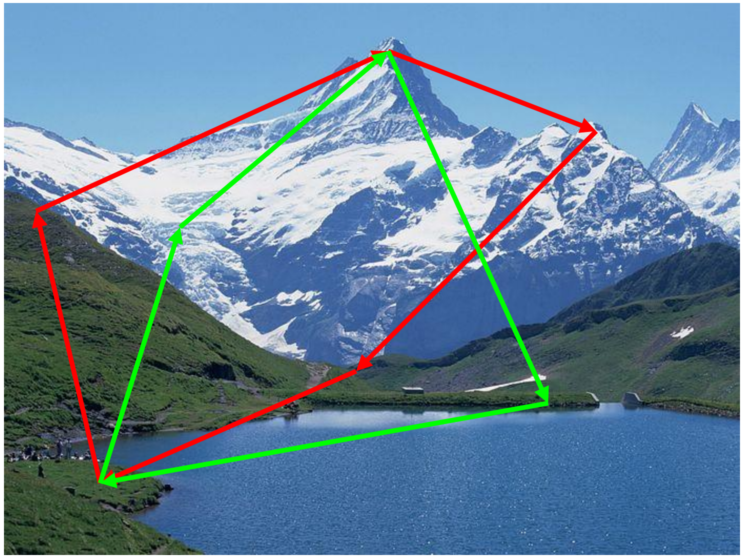
- By KCL

$$I_A = I_B + I_C + I_D$$

or $$I_A + (-I_B) + (-I_C) + (-I_D) = 0$$

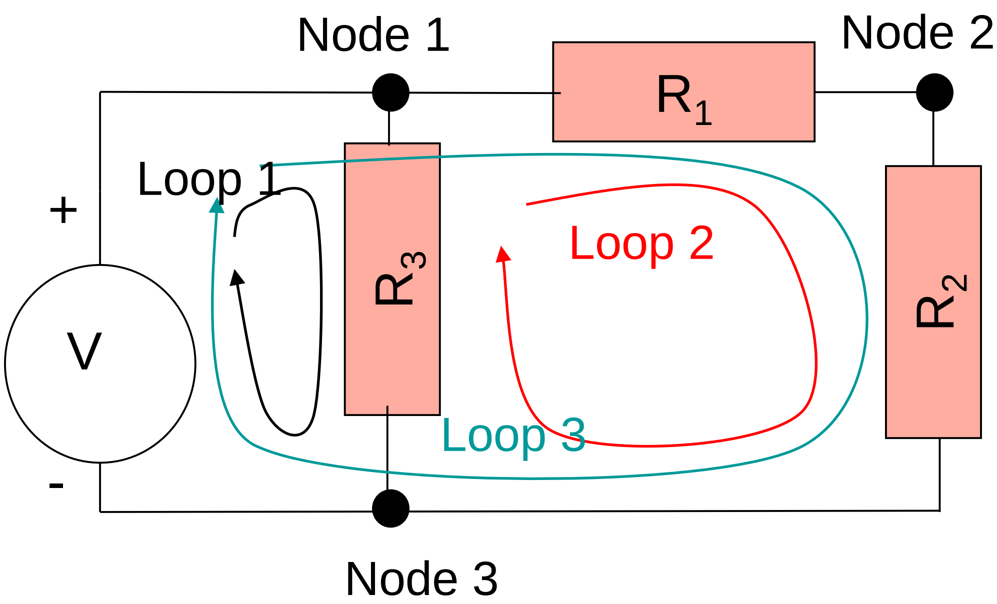
KIRCHHOFF'S VOLTAGE LAW (KVL) [1]

- The algebraic sum of all branch **voltages** around any close loop of a network is zero at all instants of time



KIRCHHOFF'S VOLTAGE LAW (KVL) [2]

- The algebraic sum of all branch voltages around any close loop of a network is zero at all instants of time
- Example:



- By KVL

L1: $V_{13} - V = 0$

L2: $V_{12} + V_{23} + V_{31} = 0$

L3: $V_{12} + V_{23} - V = 0$

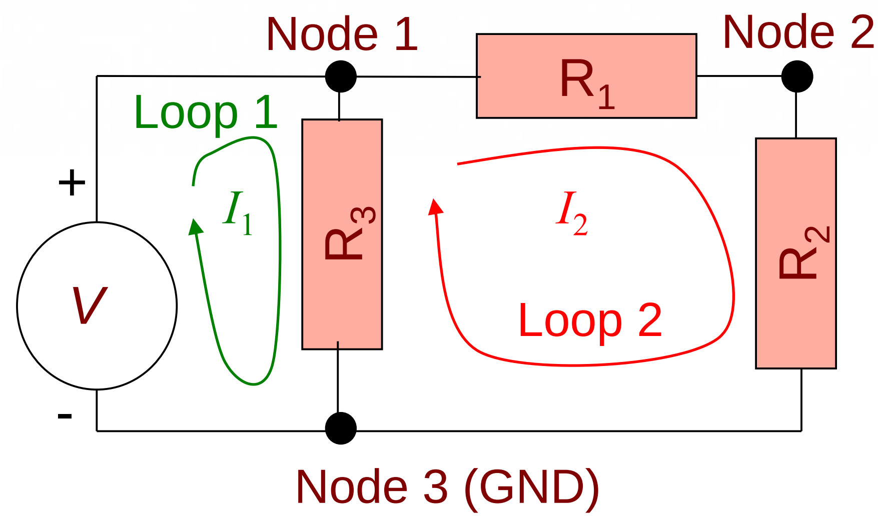
CIRCUIT ANALYSIS

- Given an electronic circuit network, circuit analysis is the process of finding the voltage across and the current through each component of the network

- We can use KCL and KVL to help us to analyze the electronic network systematically

KVL EXAMPLE

❖ Step 1: define the loops



❖ Step 2: write the loop equations

L1: $V_{13} - V = 0$

L2: $V_{12} + V_{23} + V_{31} = 0$

❖ Step 3: write all voltages in terms of loop currents

$$\begin{cases} (I_1 - I_2) R_3 - V = 0 \\ I_2 R_1 + I_2 R_2 + (I_2 - I_1) R_3 = 0 \end{cases} \Rightarrow \begin{cases} R_3 I_1 - R_3 I_2 = V \\ -R_3 I_1 + (R_1 + R_2 + R_3) I_2 = 0 \end{cases}$$

❖ All loop currents (and then node voltages) can be solved

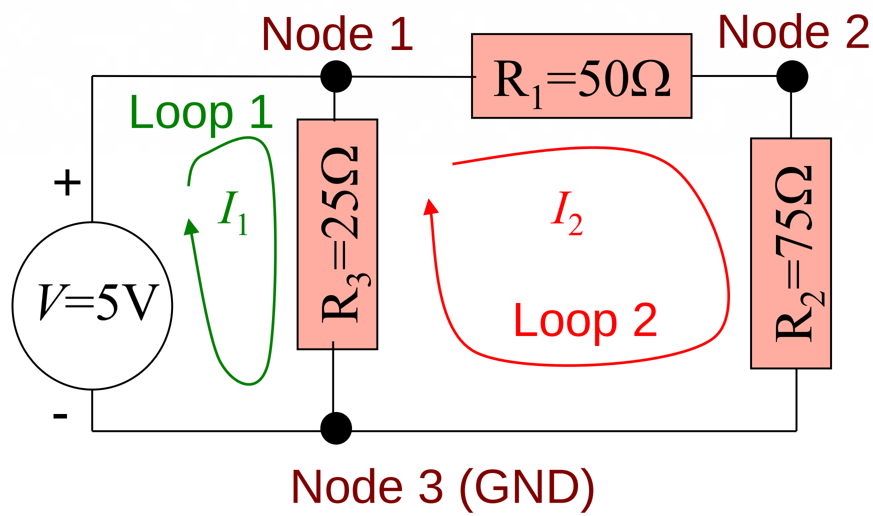
KVL: NUMERICAL EXAMPLE

- KVL

$$\begin{cases} R_3 I_1 - R_3 I_2 = V \\ -R_3 I_1 + (R_1 + R_2 + R_3) I_2 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 25 I_1 - 25 I_2 = 5 \\ -25 I_1 + (75 + 50 + 25) I_2 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} I_1 = 0.24A \\ I_2 = 0.04A \end{cases}$$



- Solutions:

$V_1 = 5$ V

$V_2 = I_2 R_2 = 0.04 \times 75 = 3$ V

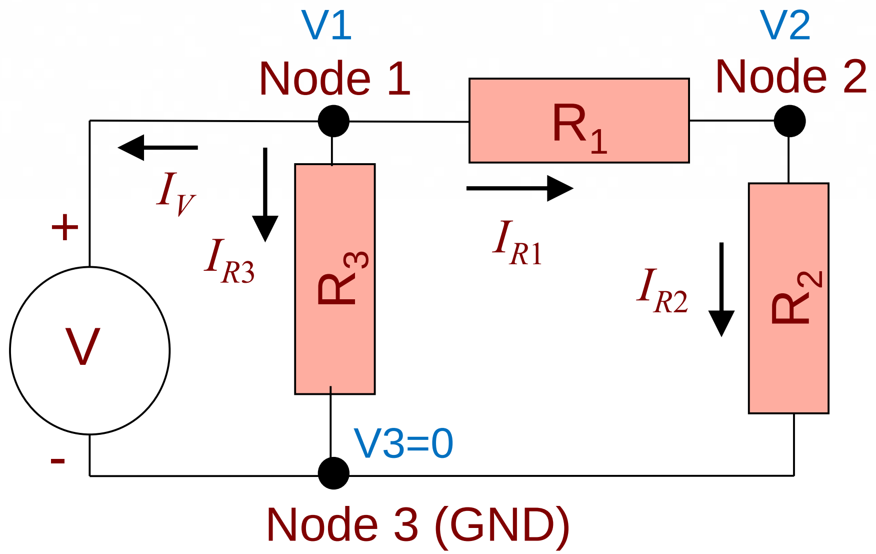
$I_{R1} = I_{R2} = I_2 = 0.04$ A

$I_{R3} = I_1 - I_2 = 0.2$ A

$I_V = I_1 = 0.24$ A

KCL EXAMPLE

- Step 1: count the nodes and define all current directions (one node can be designated as 0 or ground and ignored)



- Step 2: write the node equations

N1: $I_V + I_{R3} + I_{R1} = 0$

N2: $-I_{R1} + I_{R2} = 0$

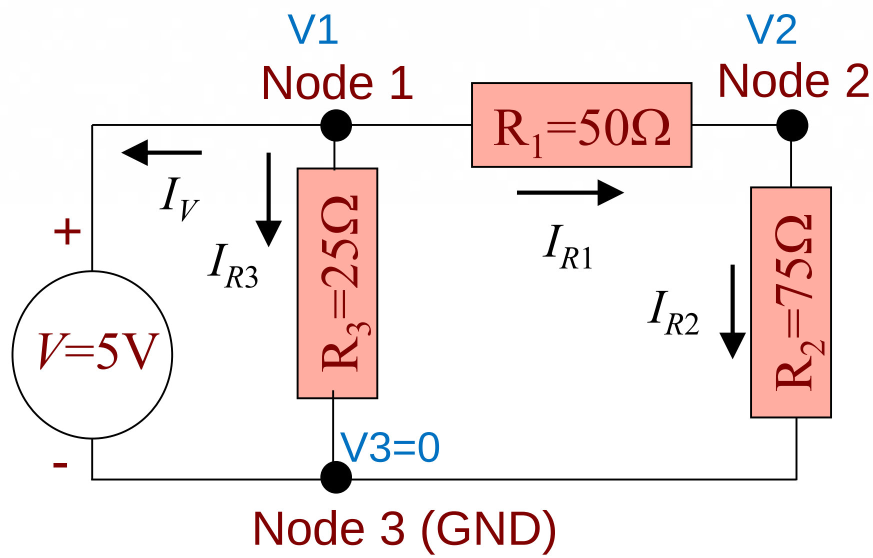
- Step 3: express branch current using node voltages

$$\begin{cases} I_V + \dfrac{V_1}{R_3} + \dfrac{V_1 - V_2}{R_1} = 0 \\ -\dfrac{V_1 - V_2}{R_1} + \dfrac{V_2}{R_2} = 0 \end{cases}$$

- Step 4: use the known voltage source voltages to eliminate some node voltages

$$\begin{cases} I_V + \dfrac{V}{R_3} + \dfrac{V - V_2}{R_1} = 0 \\ -\dfrac{V - V_2}{R_1} + \dfrac{V_2}{R_2} = 0 \end{cases} \Rightarrow \begin{cases} I_V - \dfrac{V_2}{R_1} = -\left(\dfrac{1}{R_1} + \dfrac{1}{R_3}\right)V \\ \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)V_2 = \dfrac{V}{R_1} \end{cases}$$

KCL: NUMERICAL EXAMPLE



- KCL

$$\begin{cases} I_V - \frac{V_2}{R_1} = -\left(\frac{1}{R_1} + \frac{1}{R_3}\right)V \\ \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_2 = \frac{V}{R_1} \end{cases}$$

$$\Rightarrow \begin{cases} I_V - \frac{1}{50}V_2 = -\left(\frac{1}{50} + \frac{1}{25}\right)5 \\ \left(\frac{1}{50} + \frac{1}{75}\right)V_2 = \frac{5}{50} \end{cases} \Rightarrow \begin{cases} I_V - \frac{1}{50}V_2 = -\frac{3}{10} \\ \frac{1}{3}V_2 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} I_V = -0.24A \\ V_2 = 3 \end{cases}$$

- Solutions:

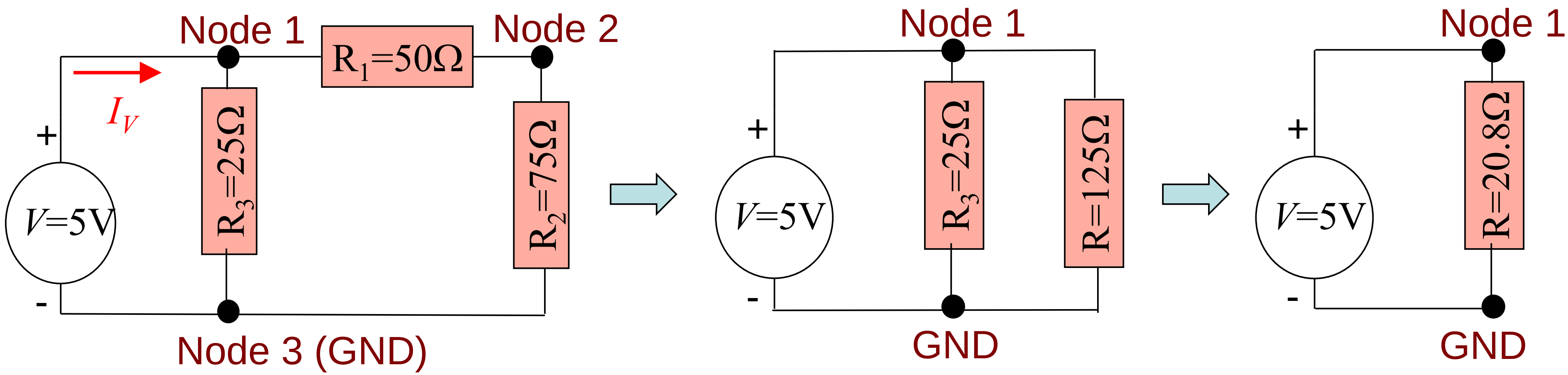
$V_1 = 5$ V $\qquad I_{R1} = (V_1 - V_2)/R_1 = 0.04$ A

$V_2 = 3$ V $\qquad I_{R2} = V_2/R_2 = 0.04$ A

$I_V = -0.24$ A $\qquad I_{R3} = V_1/R_3 = 0.2$ A

CIRCUIT SIMPLIFICATION

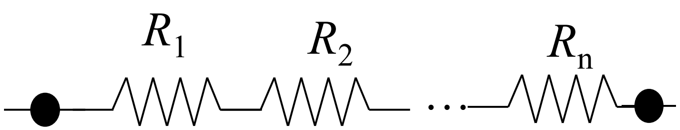
- In the given circuit, we can simplify the circuit and solve individual value without using KVL or KCL
- Example: to find I_V

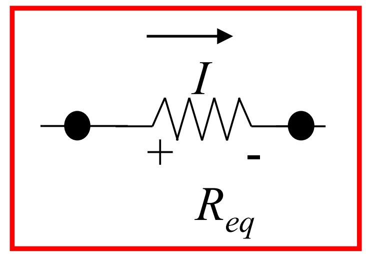


- Then apply Ohm's law to find the current

SIMPLIFICATION PROCEDURE

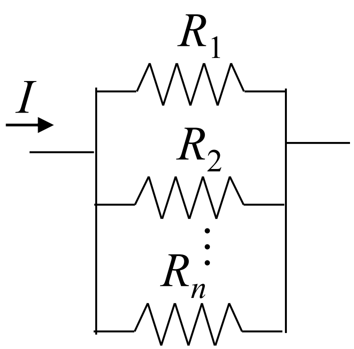
- All branch voltage in parallel with one voltage source is known
- All branch current in series with one current source is known
- For resistors in series, replace it with a resistor equal to the sum of all resistors

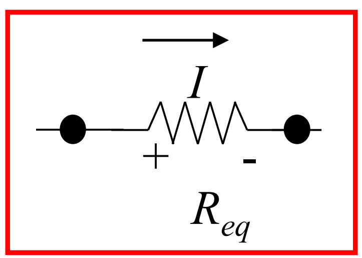




$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

- Resistors in parallel can also be combined by



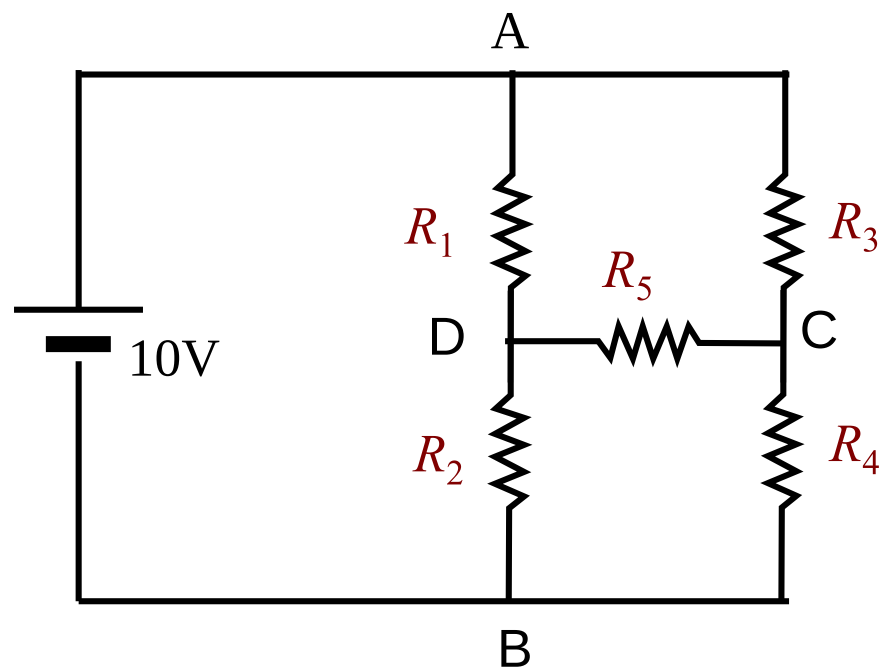


$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$

ANOTHER CIRCUIT EXAMPLE

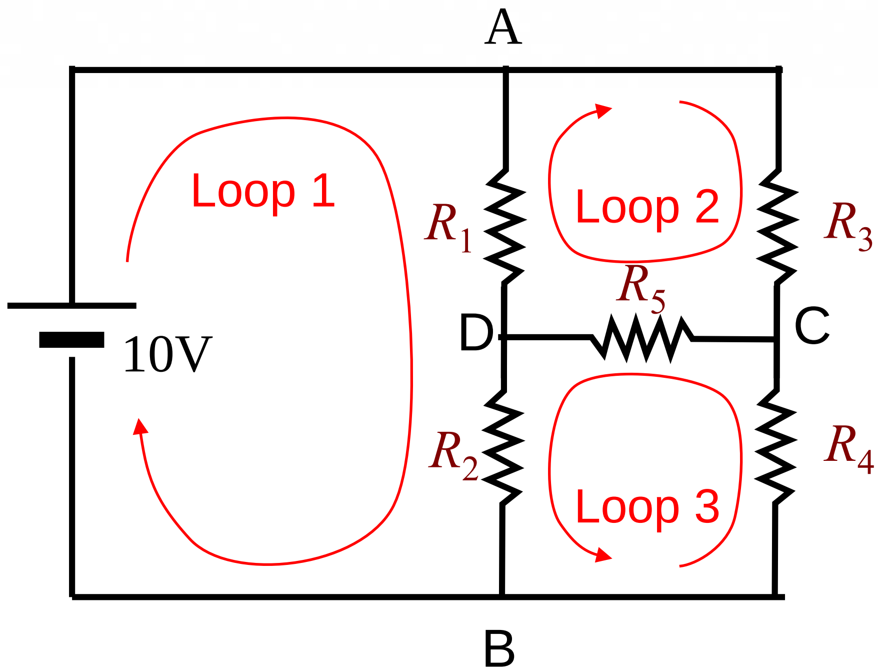
- Consider the given circuit, can you determine the total current from the 10V source by simplifying the circuit?

- How many nodes are there?
- How many branches in the circuit?



DEFINING LOOPS

- Step 1: Defining all loops



- Is there any branch not covered by the loops?
- Any other ways you want to define the loops?

WRITING LOOP EQUATIONS

❖ Step 2: Writing the loop equations

➢ Loop 1

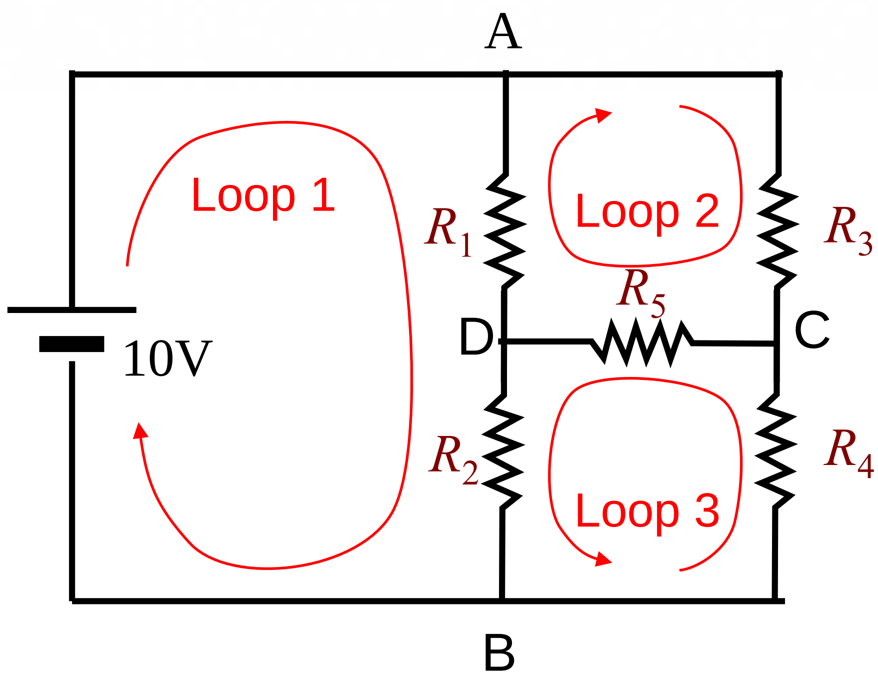
$$V_{AD} + V_{DB} - 10 = 0$$

➢ Loop 2

$$V_{AC} + V_{CD} + V_{DA} = 0$$

➢ Loop 3

$$V_{BC} + V_{CD} + V_{DB} = 0$$



CONVERTING TO LOOP CURRENTS

❖ Step 3: Express in terms of loop currents

- **Loop 1**

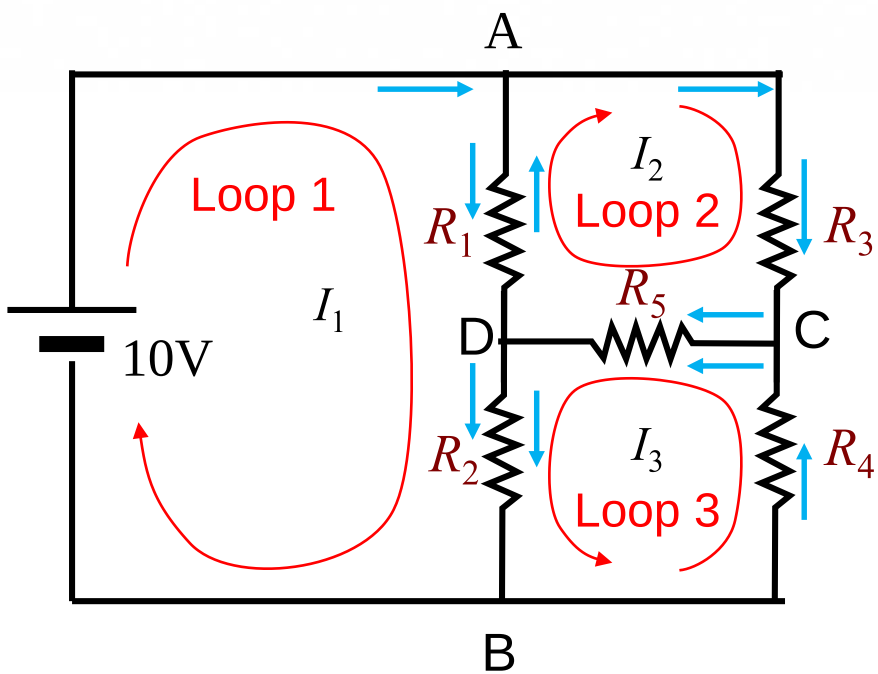
$$V_{AD}+V_{DB}-10=0$$
$$\Rightarrow (I_1-I_2)R_1+(I_1+I_3)R_2=10$$

- **Loop 2**

$$V_{AC}+V_{CD}+V_{DA}=0$$
$$\Rightarrow I_2R_3+(I_2+I_3)R_5+(I_2-I_1)R_1=0$$

- **Loop 3**

$$V_{BC}+V_{CD}+V_{DB}=0$$
$$\Rightarrow I_3R_4+(I_2+I_3)R_5+(I_1+I_3)R_2=0$$



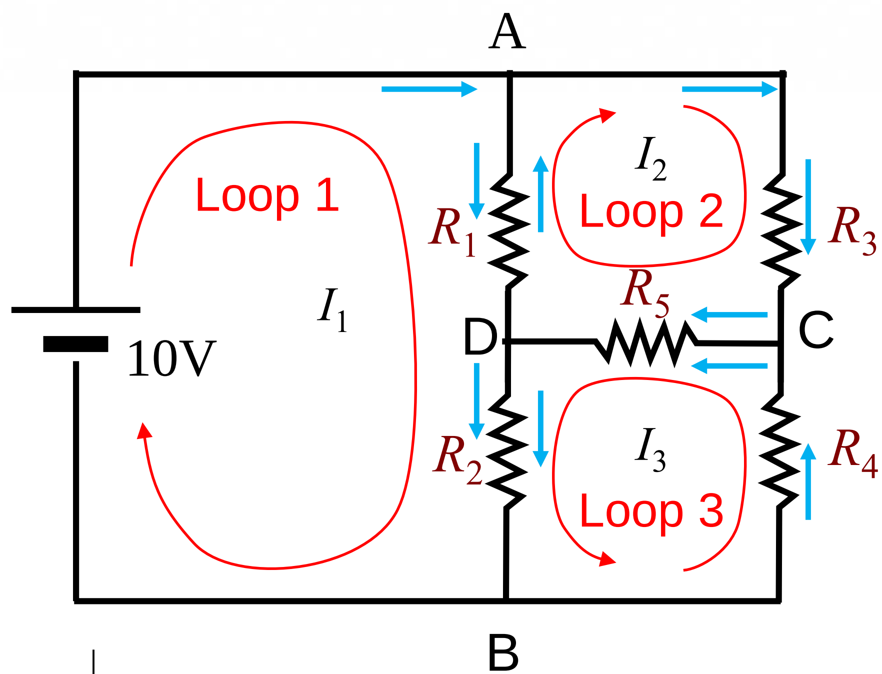
SOLVING THE SYSTEM OF EQUATIONS

❖ Rewrite the equations

$$(R_1 + R_2)I_1 - R_1 I_2 + R_2 I_3 = 10$$

$$-R_1 I_1 + (R_1 + R_3 + R_5)I_2 + R_5 I_3 = 0$$

$$R_2 I_1 + R_5 I_2 + (R_2 + R_4 + R_5)I_3 = 0$$



❖ Solve for I_1, I_2 and I_3

❖ Calculate branch currents and node voltages

$I_{R1} = I_1 - I_2$

$I_{R2} = I_1 + I_3$

$I_{R3} = I_2$

$I_{R4} = I_3$

$I_{R5} = I_2 + I_3$

$V_{AB} = 10V$

$V_{DB} = I_{R2} R_2$

$V_{CB} = -I_{R4} R_4$

SOLVING FOR VOLTAGE

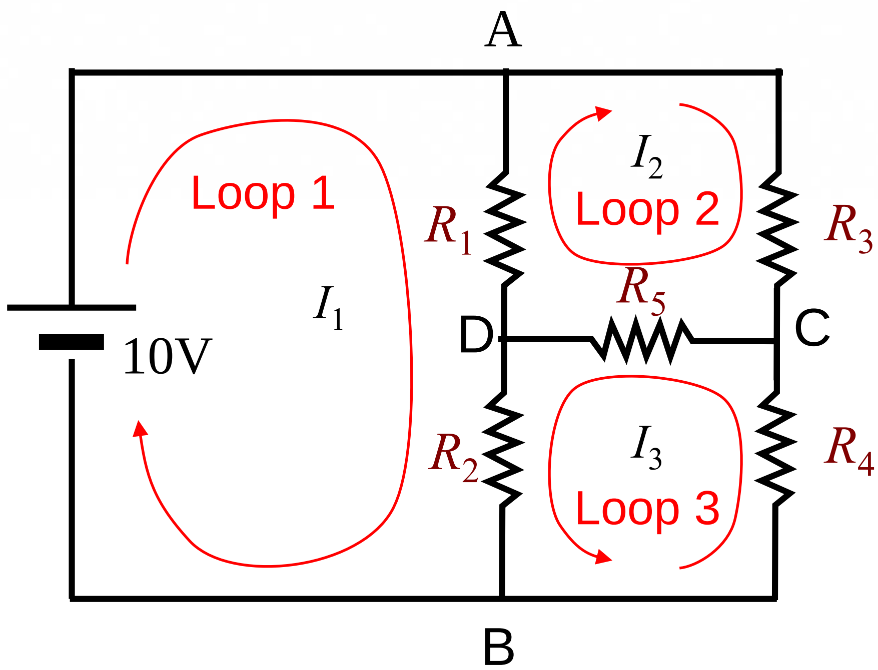
- The resistor values are given by:

  R_1=100Ω, R_2=200Ω, R_3=300Ω,

  R_4=400Ω and R_5=500Ω

- Solve for I_1, I_2 and I_3 ; we have the approximate solution of

  $I_1 = 48\text{mA}; \quad I_2 = 14\text{mA}; \quad I_3 = \text{-}15\text{mA};$

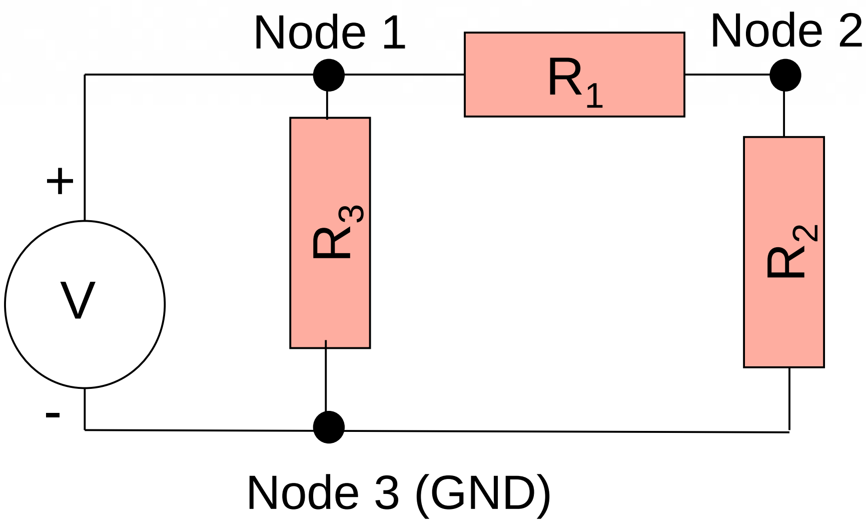


CAD TOOLS

- If the network has many components, the number of variables and equations too large to solve by hand calculation
- Have to rely on computer software program to solve these complex systems
- We call this a computer-aided design (CAD) tool
- You will learn how to use CAD tools in other design courses

LECTURE SUMMARY

- Kirchhoff's Current Law
  - the algebraic sum of all branch currents entering and leaving a node is zero at all instants of time
- Kirchhoff's Voltage Law
  - the algebraic sum of all branch voltages around any close loop of a network is zero at all instants of time



- We have demonstrated how to use KVL and KCL to solve a simple circuit together

NEXT LECTURE

- Sensors
- Amplification of sensor signals
- Lab Midterm Review

QUESTIONS?